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A novel multi-dictionary framework with global sensing matrix design for compressed sensing

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ABSTRACT

In this paper, a new compressed sensing (CS) system is proposed to reduce computational burden of dictionary learning and improve reconstruction accuracy. The proposed CS system employs a novel framework which contains multiple dictionaries. In multi-dictionary framework, the whole training dataset is divided into multiple subdatasets for optimizing multiple dictionaries. Dictionary learning process can be accelerated due to the parallel computation and the reduction of training dataset size. Each dictionary can get an image (called snapshot) independently with the same measurements in the image reconstruction process. These snapshots will be fused to be one image with averaging strategy. In order to keep the measurement size of the proposed CS system same as that of traditional CS system and improve reconstruction accuracy, a new method of designing global sensing matrix for multi-dictionary framework is also explored. Experiments demonstrate the effectiveness of new framework and the method to design global sensing matrix. Compared with other CS systems, the proposed CS system shows a superior performance for real images.

1. Introduction

Compressed sensing (CS) has attracted lots of attention since its introduction [1–4]. At first, CS is a mathematical framework that recovers sparse signals from the projection of original high-dimensional signals accurately. That is, a sparse signal vector $\mathbf{x} \in \mathbf{R}^{N\times 1}$ can be recovered from the measurement vector $\mathbf{y} \in \mathbf{R}^{M\times 1}$ $(M \ll N)$ through a chosen sensing matrix $\mathbf{\Phi} \in \mathbf{R}^{M\times N}$:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{x}. \tag{1}$$

Since $M \ll N$, (1) is an under-determined problem, which may have more than one solution. Therefore, the sparsity of original signal $\mathbf{x}$ can be an appropriate constraint in the CS system to make the solution unique. For convince, the $\mathcal{L}_0$ norm $(\|\cdot\|_0)$ which is used to count the number of non-zero entries is employed for measuring the sparsity of vectors. However, when signal $\mathbf{x}$ is dense in the real world, it should be represented sparsely by an over-complete dictionary $\mathbf{\Psi} \in \mathbf{R}^{N\times L}$:

$$\mathbf{x} = \mathbf{\Psi}\boldsymbol{\theta}, \tag{2}$$

where $\boldsymbol{\theta} \in \mathbf{R}^{L\times 1}$ is the sparse coefficient of signal $\mathbf{x}$ and the sparse constraint is satisfied by $\boldsymbol{\theta}$. That is, the signal can be K-sparse if $\|\mathbf{x}\|_0 \le K$ or $\|\boldsymbol{\theta}\|_0 \le K$.

There may exist noise or interference in practical. The approximate representation can be obtained by minimizing the error of sparse representation. Thus, a sparsifying dictionary $\mathbf{\Psi}$ for $\mathbf{x}$ can be gotten through the formulation:

$$\min_{\boldsymbol{\theta},\mathbf{\Psi}} \| \mathbf{x} - \mathbf{\Psi}\boldsymbol{\theta} \|_2^2 . \tag{3}$$

Let the matrix $\mathbf{X} = [\mathbf{x}_1, \cdots, \mathbf{x}_W]$ be a set of samples from a class of signals to be considered [10]. $\mathbf{\Psi}$ can be obtained through the formulation:

$$\min_{\mathbf{\Theta},\mathbf{\Psi}} \| \mathbf{X} - \mathbf{\Psi}\mathbf{\Theta} \|_F^2, \tag{4}$$

where $\mathbf{\Theta} = [\boldsymbol{\theta}_1, \cdots, \boldsymbol{\theta}_W]$, $\mathbf{\Theta}$ is the aggregation of the sparse coefficients. The reasonable method to solve (3) or (4) is alternative iteration, i.e. sparse coding [5–7] and dictionary learning [8–12]. Among plenty of practical dictionary learning algorithms, K-singular value decomposition (K-SVD) [9] is widely used, and orthogonal matching pursuit (OMP) is generally employed for sparse coding [5,6].

With (1) and (2), the framework of the CS can be rewritten as:

$$\mathbf{y} = \mathbf{\Phi}\mathbf{\Psi}\boldsymbol{\theta} = \mathbf{D}\boldsymbol{\theta}, \tag{5}$$

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in which, $\mathbf{D} \in \mathbf{R}^{M \times L}$ is the equivalent dictionary.

Signal can be recovered from its measurements with high probability if the following restricted isometry property (RIP) holds [3,10,13], i.e. with constant $0 \le \delta < 1$,

$$(1-\delta) \parallel \boldsymbol{\theta} \parallel_2^2 \le \parallel \mathbf{D}\boldsymbol{\theta} \parallel_2^2 \le (1+\delta) \parallel \boldsymbol{\theta} \parallel_2^2$$

holds for all K-sparse vectors $\boldsymbol{\theta}$. However, the RIP is not tractable, so the mutual coherence $\mu(\mathbf{D})$ that represents the worst case coherence between any two columns of $\mathbf{D}$ is preferable [14,16]:

$$\mu(\mathbf{D}) \triangleq \max_{1 \le i \ne j \le L} \frac{| \mathbf{D}(:,i)^T \mathbf{D}(:,j) |}{\parallel \mathbf{D}(:,i) \parallel_2 \parallel \mathbf{D}(:,j) \parallel_2}, \tag{6}$$

where $\mathbf{D}(:,i)$ and $\mathbf{D}(:,j)$ represent the ith column and the jth column of $\mathbf{D}$, respectively.[1] According to [14], the K-sparse signal can be exactly reconstructed as long as

$$\mu(\mathbf{D}) < \frac{1}{2K-1}.$$

For this purpose, the optimal cost function of sensing matrix $\boldsymbol{\Phi}$ with target Gram matrix $\mathbf{G}_t$ is defined as:

$$\boldsymbol{\Phi} = \arg\min_{\tilde{\boldsymbol{\Phi}}} \parallel \mathbf{G}_t - \mathbf{D}^T\mathbf{D} \parallel_F^2 \quad \text{s.t.} \mathbf{D} = \tilde{\boldsymbol{\Phi}}\boldsymbol{\Psi}, \tag{7}$$

where $\mathbf{G}_t$ can be an identical matrix or relaxed equiangular tight frame (ETF) [10,15]. The first work that optimizes the sensing matrix is [16] and the average mutual coherence, $\mu_{av}(\mathbf{D})$, was used. When $\mathbf{G}_t$ is an ETF, the performance of sensing can be improved [17–19].

Many approaches [10,15,20,24] design the sensing matrix $\boldsymbol{\Phi}$ based on (7). Their purpose is to make Gram matrix $\mathbf{G} \triangleq \mathbf{D}^T\mathbf{D}$ and target Gram matrix $\mathbf{G}_t$ as close as possible. The method of gradient descent [17,25] is widely used to optimize sensing matrix with a fixed dictionary.

The idea of employing multiple algorithms is employed in many fields [26–31]. In this paper, a new CS system with multiple dictionaries and global sensing matrix is proposed to reduce computational burden of dictionary learning [32,33] and improve reconstruction accuracy [34]. The main idea of this proposed CS system is using multiple dictionaries to reconstruct signal with a global sensing matrix.

The main contributions of this work are summarized in the following three folds:

1. A novel framework which contains multiple dictionaries is proposed for CS system;
2. The method to design global sensing matrix for multi-dictionary framework is explored;
3. Experiments are carried out to show the advantages of these two methods and the superior performance of the proposed CS system.

The rest of this paper is organized as follows. In Section 2, we will introduce the proposed CS system in detail. In Section 3, some experiments are designed to demonstrate the advantages of these two methods for the proposed CS system. In Section 4, the proposed CS system is compared with other CS systems and its superior performance is shown. In Section 5, some conclusions and the intending works are given.

2. The new compressed sensing system with multiple dictionaries and global sensing matrix

In this section, we will describe the new CS system. In the Section 2.1, the details of multi-dictionary framework is introduced. The approach of designing global sensing matrix is proposed in the Section 2.2. The computational complexity of the proposed CS system is demonstrated in Section 2.3.

[1] Throughout this paper, MATLAB notations are used.

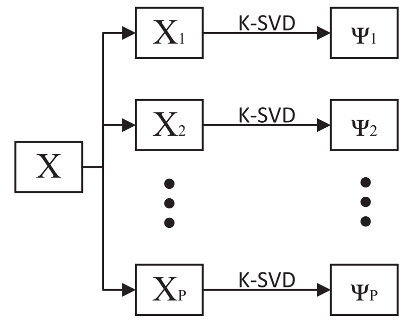

Fig. 1. The process of training multiple dictionaries.

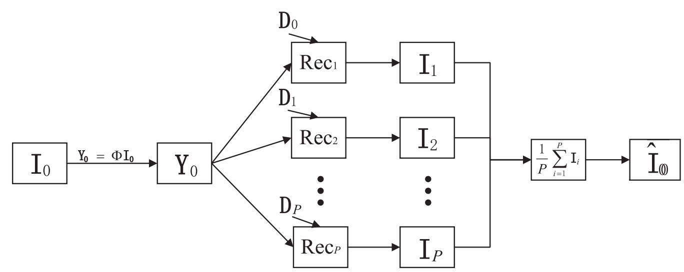

Fig. 2. The framework of image reconstruction.

2.1. The multi-dictionary framework

The multi-dictionary framework contains two parts, training multiple dictionaries and reconstructing image with these dictionaries.

2.1.1. Training multiple dictionaries

Supposed we have the training dataset $\mathbf{X} \in \mathbf{R}^{N \times W}$. The whole dataset will be divided into P subdatasets randomly at first. Then, each subdataset, $\mathbf{X}_k \in \mathbf{R}^{N \times (W/P)} (k = 1, \cdots, P)$, is used to train one dictionary. In this paper, the classical algorithm, K-SVD [9], is employed to train dictionary. The process of training multiple dictionaries is shown in Fig. 1.

Compared with traditional dictionary learning which usually uses the whole training dataset to train one dictionary, our proposed method can reduce the training dataset size of single dictionary and employ the parallel computation to train multiple dictionaries simultaneously.

The pseudo-code of training multiple dictionaries is described as Algorithm 1.

Algorithm 1 Training multiple dictionaries.

Input: Training dataset $\mathbf{X} \in \mathbf{R}^{N \times W}$.
Initial: Divide the whole training dataset $\mathbf{X}$ into P subdatasets randomly,i.e., $\mathbf{X}_k \in \mathbf{R}^{N \times (W/P)}, (k = 1, \cdots, P)$.
for $(i = 1 : P)$ **do**
1. Select the subdataset $\mathbf{X}_i$ as the training dataset;
2. Train the dictionary $\boldsymbol{\Psi}_i$ with K-SVD algorithm.
end for
Output: Multiple dictionaries $\{\boldsymbol{\Psi}_k\}_{k=1}^P$.

2.1.2. Reconstructing image with multiple dictionaries

Here, we will introduce and analyze the method to reconstruct one image with multiple dictionaries. Fig. 2 shows the reconstruction process. It should be noted that all dictionaries use the same measurements $\mathbf{Y}_0$. The measurements $\mathbf{Y}_0 = [\mathbf{y}_1, \cdots, \mathbf{y}_W]$ and can be calculated according to (1). The method of designing global sens-