Automatica 49 (2013) 2906–2910

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Stochastic system controller synthesis for reachability specifications encoded by random sets ☆

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ARTICLE INFO

Article history:
Received 9 March 2012
Received in revised form
27 May 2013
Accepted 11 June 2013
Available online 12 July 2013

Keywords:
Hybrid systems
Stochastic systems
Reachability
Safety
Target hitting
Optimal control
Dynamic programming

ABSTRACT

We consider a reach–avoid specification for a stochastic hybrid dynamical system defined as reaching a goal set at some finite time, while avoiding an unsafe set at all previous times. In contrast with earlier works which consider the target and avoid sets as deterministic, we consider these sets to be probabilistic. An optimal control policy is derived which maximizes the reach–avoid probability. Special structure on the stochastic sets is exploited to make the computation tractable for large space dimensions.

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1. Introduction

Stochastic Hybrid System (SHS) models provide a framework for the analysis and design of complex dynamical systems. While reachability and safety analysis for deterministic systems have been well-studied (Aubin, 1991; Lygeros, 2004; Tomlin, Mitchell, Bayen, & Oishi, 2003), these problems for SHS are recently being explored. In the continuous time setting, early contributions to SHS theory include the works of Davis (1993) and Ghosh, Arapostathis, and Marcus (1997), with (Bujorianu & Lygeros, 2003) establishing a theoretical foundation for the measurability of events for reachability problems. Because technical issues such as measurability are easier to handle in the discrete time setting, Discrete Time Stochastic Hybrid System (DTSHS) models have recently attracted considerable attention. In particular, probabilistic reachability of DTSHS has recently been addressed in Abate, Prandini, Lygeros, and Sastry (2008), Ramponi, Chatterjee, Summers, and Lygeros (2010), Summers, Kamgarpour, Tomlin, and Lygeros (2011) and Summers and Lygeros (2010) based on a theoretical foundation for discrete time stochastic optimal control (Bertsekas & Shreve, 2007).

In Summers et al. (2011), we extended the results of Abate et al. (2008) and Summers and Lygeros (2010) on verification of stochastic hybrid systems to cope with the existence of uncertainty in the reachability specifications themselves. In particular, we considered the problem of maximizing the probability that a system trajectory will hit a target set while avoiding an unsafe set where the safe set was allowed to be random and time-varying. The proof was omitted due to space.

Since the work in Summers et al. (2011) set up the theoretical framework for several applications, we dedicate this paper mainly to the proof of the result in Summers et al. (2011). At the same time, we consider a slightly more general framework in which both the safe and target sets are random, time-varying and parametrized by stochastic processes. While our emphasis is on the theory, we demonstrate the applicability of the framework with a small scale example. Please refer to Summers et al. (2011) for an example in air traffic scenario and Kariotoglou, Raimondo, Summers, and Lygeros

☆ This work was partially supported by the European Commission under the project MoVeS, FP7-ICT-2009-257005, by ETH Zurich under Grant ETH-12 09-2, by the AFOSR under grant FA9550-06-1-0312 and by the Natural Sciences and Engineering Research Council of Canada (NSERC). Sean Summers and Maryam Kamgarpour contributed equally to this work. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Bart De Schutter under the direction of Editor Ian R. Petersen.

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0005-1098/$ – see front matter © 2013 Elsevier Ltd. All rights reserved.
http://dx.doi.org/10.1016/j.automatica.2013.06.016

(2011) for applications in camera networks, both of which are of larger space dimensions.

2. Stochastic sets for reach–avoid problem

Let the state space of the discrete time stochastic hybrid system DTSHS, be represented by a hybrid set $X := \bigcup_{q\in\mathcal{Q}}\{q\}\times\mathbb{R}^{n(q)}$, where the discrete state space is $\mathcal{Q} := \{1, 2, \ldots, m\}$, $m \in \mathbb{N}$ and the map $n : \mathcal{Q} \to \mathbb{N}$ assigns to each discrete state $q \in \mathcal{Q}$ the dimension of the continuous state space $\mathbb{R}^{n(q)}$. We endow the set $\mathcal{Q}$ with the discrete topology, Euclidean spaces with the Euclidean topology, and the set X with the corresponding product topology. As usual, the DTSHS as defined is more generally characterized as a Markov Decision Process. Throughout, let $\mathcal{B}(\cdot)$ denote the Borel σ-algebra of the topological space.

We define a parametrized stochastic set-valued process as follows. Let $Y \subset \mathbb{R}^o$ for $o \in \mathbb{N}$ denote a parameter space. For $k = 0, 1, 2, \ldots, N$, let G_k be a Borel-measurable stochastic kernel on Y given Y, $G_k : \mathcal{B}(Y) \times Y \to [0, 1]$, which assigns to each $y \in Y$ a probability measure $G_k(\cdot|y)$ on the Borel space $(Y, \mathcal{B}(Y))$. A stochastic set is henceforth defined as a Borel measurable set-valued function $\gamma : Y \to \mathcal{B}(X)$ together with a Markov process $\{y_k\}_{k\in\mathbb{N}}$ in the parameter space Y with transition probability function $G_k : \mathcal{B}(Y) \times Y \to [0, 1]$.

Let A be a compact Borel set representing the input space and define the controlled transition probability function $Q : \mathcal{B}(X) \times X \times A \to [0, 1]$, that is, $Q(\cdot|x, a)$ assigns a probability measure on $\mathcal{B}(X)$ for each hybrid state $x \in X$ and input $a \in A$. By appropriate selection of Q one can encode a wide range of hybrid dynamics, including autonomous and forced transitions. Let $\bar{x} = (x, y)$ be the augmented state in $\bar{X} = X \times Y$, the augmented state space. Further, let us define the stochastic transition kernels $\bar{Q}_k : \mathcal{B}(\bar{X}) \times \bar{X} \times A \to [0, 1]$:

$$\bar{Q}_k(d\bar{x}'|\bar{x}, a) = Q(dx'|x, a)G_k(dy'|y).$$

We call the resulting stochastic process an Augmented Discrete-Time Stochastic Hybrid System (ADTSHS) $\bar{\mathcal{H}}$. The ADTSHS kernel is time varying, due to the kernel G_k, factorizes to a product of two stochastic kernels on the hybrid state and the parameter spaces, and depends on the control variable $a \in A$ only through the DTSHS kernel. The latter two properties are critical to the numerically tractable algorithm provided here. We define a Markov policy for this process as follows.

Definition 1. A Markov policy for $\bar{\mathcal{H}}$ is a sequence $\mu = (\mu_0, \mu_1, \ldots, \mu_{N-1})$ of universally measurable maps $\mu_k : \bar{X} \to A$, $k = 0, 1, \ldots, N-1$. The set of all admissible Markov policies is denoted by $\bar{\mathcal{M}}$.

Given a Markov policy $\mu \in \bar{\mathcal{M}}$ and initial state $(x_0, y_0) \in X \times Y$, the execution of the augmented process denoted by $\{(x_k, y_k), k = 0, 1, \ldots, N\}$ is a stochastic process defined on the canonical sample space $\Omega := \bar{X}^{N+1}$, endowed with its product σ-algebra $\mathcal{B}(\Omega)$. The probability measure $P^{\mu}_{(x_0,y_0)}$ on Ω is uniquely defined by the transition kernels $\bar{Q}_k$, the Markov policy $\mu \in \bar{\mathcal{M}}$, and the initial state $(x_0, y_0) \in \bar{X}$ (Bertsekas & Shreve, 2007).

Consider the stochastic kernels $G_k : \mathcal{B}(Y) \times Y \to [0, 1]$, the parameter process $\{y_k\}$, for $k = 0, 1, \ldots, N$ distributed according to these kernels, along with two functions $\gamma : Y \to \mathcal{B}(X)$ and $\gamma' : Y \to \mathcal{B}(X)$, such that $\gamma(y) \subseteq \gamma'(y)$, $\forall y \in Y$. Define $K_k := \gamma(y_k)$ and $K'_k := \gamma'(y_k)$ as stochastic target and safe sets respectively.

Our goal is to evaluate and subsequently maximize the probability that the execution of the Markov control process $\{x_k\}_{k=0,1,\ldots,N}$ will reach the target set K_k at some time in the horizon while remaining in K'_k, at all prior times. The probability that the system initialized at $x_0 \in X$, $y_0 \in Y$, with control policy $\mu \in \bar{\mathcal{M}}$ reaches K_k while avoiding $X \setminus K'_k$ for all $k = 0, 1, \ldots, N$ is

$$r^{\mu}_{(x_0,y_0)}(\gamma, \gamma') := P^{\mu}_{(x_0,y_0)}\{\exists j \in [0, N] : x_j \in \gamma(y_j) \wedge \forall i \in [0, j-1]\, x_i \in \gamma'(y_i) \setminus \gamma(y_i)\}. \quad (1)$$

Our first observation is that the reachability probability may be computed as an expectation of event. For this, let $\bar{K} = \{(x, y) \in X \times Y \mid x \in \gamma(y)\}$ and $\bar{K}' = \{(x, y) \in X \times Y \mid x \in \gamma'(y)\}$. The reach–avoid probability (1) is

$$E^{\mu}_{\bar{x}_0}\left[\mathbf{1}_{\bar{K}}(\bar{x}_0) + \sum_{j=1}^{N}\left(\prod_{i=0}^{j-1}\mathbf{1}_{\bar{K}'\setminus\bar{K}}(\bar{x}_i)\right)\mathbf{1}_{\bar{K}}(\bar{x}_j)\right], \quad (2)$$

where $\bar{x}_k = (x_k, y_k)$ and we work under the convention that $\prod_{i=k}^{j}(\cdot) = 1$ for $k > j$, for $D \subset X$, $\mathbf{1}_D : X \to \{0, 1\}$ denotes its indicator function and $E^{\mu}_{\bar{x}_0}$ is the expectation with respect to the probability measure $P^{\mu}_{(x_0,y_0)}$.

Our control objective is as follows. Given an ADTSHS $\bar{\mathcal{H}}$, with stochastic set parameters $y \in Y$, and set-valued maps γ and γ', $\gamma'(y) \subseteq \gamma(y)$ for all $y \in Y$ representing target and safe sets respectively:

(1) Compute the maximum reach–avoid probability $r^{*}_{(x_0,y_0)}(\bar{K}, \bar{K}') := \sup_{\mu\in\bar{\mathcal{M}}} r^{\mu}_{(x_0,y_0)}(\bar{K}, \bar{K}')$, $\forall x_0 \in X$.
(2) Find an optimal Markov policy $\mu^* \in \bar{\mathcal{M}}$, if it exists, such that $r^{*}_{x_0}(\bar{K}, \bar{K}') = r^{\mu^*}_{x_0}(\bar{K}, \bar{K}')$, $\forall x_0 \in X$.

Note that deterministic Markov policies are possibly a restriction. We are working on extensions of the results to randomized policies.

From Eq. (2), observe that the reach–avoid problem with stochastic sets is transformed into a reach–avoid problem with deterministic sets in an extended state space. Hence, reach–avoid methods developed for deterministic safe and target sets (Summers & Lygeros, 2010) can be applied. However, these methods are computationally intractable for any hybrid and parameter spaces with a large combined dimension due to the Curse of Dimensionality. Here, we focus on a special case of the above problem that greatly reduces the computational burden.

3. A tractable case of the reach–avoid problem

Assume that the Markov parameters describing the stochastic sets are given as, or can be fairly approximated by, independent probability distributions. That is, let $G_k(dy_k|y_{k-1}) = G_k(dy_k)$. Due to the independence of the probability distribution G_k from the parameter y_k, without loss of generality, we consider the Markov policy also being independent from the parameter y_k. Thus, we consider the Markov policy as sequence of universally measurable maps $\mu_k : X \to A$, $k = 0, 1, \ldots, N$. Let $\mathcal{M}$ denote the set of all such policies. Note that due to this assumption, the closed loop transition kernels $\bar{Q}_k(\cdot|\bar{x}_k, \mu_k(\bar{x}_k))$ become equivalent to the product of two decoupled transition kernels $Q(\cdot|x_k, \mu_k(x_k))$ and $G_k(y_k)$. We use $Q^{\mu_k}(dx'|x_k)$ as a short-hand notation for $Q(dx'|x_k, \mu_k(x_k))$ for $k = 0, 1, \ldots, N-1$.

Since the initial parameter state y_0 of the stochastic set is assumed known, we define $G_0(dy_0|y_{0-1}) := \delta_{y_0}(dy_0)$. For the set valued maps γ and γ', with $\gamma(y) \subseteq \gamma'(y)$ for all $y \in Y$, we define the following *covering functions*:

$$p_{K_k}(x) = \int_Y \mathbf{1}_{\gamma(y_k)}(x)G_k(dy_k) = E\left[\mathbf{1}_{\gamma(y_k)}(x)\right],$$

$$p_{K'_k}(x) = \int_Y \mathbf{1}_{\gamma'(y_k)}(x)G_k(dy_k) = E\left[\mathbf{1}_{\gamma'(y_k)}(x)\right].$$

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