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Noise effect on signal quantization in an array of binary quantizers

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ABSTRACT

The effect of noise is examined in both the uniform quantizer and a stochastic quantizer. The stochastic quantizer is an array of identical binary quantizers combined with a linear decoder. In uniform quantization scenario, we find that noise cannot help decrease mean square error (MSE) distortion. However, in the array of binary quantizers with identical thresholds, noise may play a positive role, and stochastic resonance (SR) can be observed. First, based on MSE distortion, the optimal noise in the array is derived by Gateaux differential, which is shown to be a uniform noise. And then, some other noises, including uniform noise with zero mean, Gaussian noise, Laplacian noise, and discrete noise, are considered for comparison. In the case that the granular region is fixed, the quantization performances induced by those noises are shown, and the SR effects are discussed. Furthermore, when the granular region is adjustable, quantization performance may be better. Especially, the MSE distortion, in the optimal noise case, will approximate zero as the rate becomes high. At last, some further studies are discussed, which may extend our results.

1. Introduction

In some nonlinear systems, random noise may significantly enhance system performance. That phenomenon is described as stochastic resonance (SR), or generally, stochastic facilitation (SF) [1]. In recent years, the noise effects on signal processing and information transmission have been extensively discussed in SR literature [2–6].

Quantization, as a lossy source coding, can also benefit from randomization or noise. When random noise is introduced, the quantizer becomes a randomized one, and the signal is quantized stochastically. Particularly, Saldi et al. have stated that the random coding in Shannon's rate-distortion theorem can be viewed as randomized quantizers [7], which highlights the significance of stochastic quantization in lossy compression.

In the stochastic quantization scheme, the statistical property of quantization error may be purposely controlled by additive noise. Wannamaker et al. have investigated such randomized quantizers, which are called dithered quantizers, in detail [8]. When a suitable noise is employed, e.g., noise with a triangular probability density function (pdf), the moments of quantization error can be independent of the input signal. Such properties are essential for the purpose of audio signal quantization. Moreover, if the additive noise can be subtracted from the output of the quantizer, the randomized system is named subtractively dithered quantizer [9]. Using the subtractive dithering, Li et al. preserved the probability distribution of input signal and enhanced the perceptual quality of audio and video coding [10]. Later, Saldi et al. considered a general problem in Ref. [7]. They have shown the existence of an optimal randomized quantizer under the constraint that the reconstructed signal has a given distribution. In addition, Akyol and Rose extended the conventional dithered quantization to a non-uniform one [11]. They proposed the optimal non-uniform randomized quantizer subject to the uncorrelated error constraint. It has been shown that for a Gaussian source, such a quantizer outperforms the conventional dithered one.

On the other hand, noise may assist in decreasing the average distortion in some stochastic quantization scenario. Especially in the system of parallel threshold devices [12,13], noise usually shows suprathreshold stochastic resonance (SSR) effect. That is to say, signal-to-quantization-noise ratio, or mean square error (MSE) distortion, is optimized at some nonzero noise intensity. In Ref. [14], McDonnell et al. discussed SSR effect of various types of noise and showed stochastic quantization efficiency by considering different decoding methods. Xu et al. have also paid attention to the decoding of signal quantization [15]. They proposed an optimal weighted decoding method for stochastic quantization in an array

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of binary quantizers. Measured by MSE distortion, the SSR effect in the optimal weighted decoding is found to be better than that in linear decoding. Later, the authors investigated the weighted decoding scheme again [16], but elements in the SSR model are nonlinear sensors instead of binary quantizers. In addition, quantization with parallel channels has been studied by Goyal, too [17]. In that case, uniform quantizers are concatenated in parallel and random dither is added to the threshold of each quantizer. As Goyal has demonstrated, distortion of the randomly-designed quantizer exceeds the distortion of the optimal quantizer just by a constant factor.

As stated above, noise sometimes has the potential to improve quantization performance. Furthermore, it is of significance to gain the utmost improvement induced by noise. In this paper, when quantization performance is quantified by MSE distortion, we attempt to find the optimal noise which leads to the minimal distortion. Based on the existing results for some given noises, e.g., Refs. [13,14,18], this paper will consider an array of identical binary quantizers combined with a linear decoder as the quantization system, and then explore the optimal noise in the framework. Meanwhile, we will study the SR (SSR) effects in our quantization system. The rest of this paper is organized as follows. Section 2 presents a review of the classical uniform quantizer. Performance of the quantizer dithered with additive noise is also analyzed. In Section 3, in linear decoding scenario, the optimal noise is derived to minimize the MSE distortion in an array of binary quantizers. The corresponding optimal quantization performance is explicitly displayed in Section 4. For comparison, SR (SSR) effects of various suboptimal noises are also shown. In Section 5, conclusion and further discussion are presented.

2. Review of classical uniform quantizer

A general quantizer Q consists of an encoder and a decoder [19].

- Encoder α: It assigns an index i to the signal x, i.e., $i = \alpha(x)$. For a quantizer with the number of quantization levels M, the set of indices is $\mathcal{I} = \{0, 1, \dots, M-1\}$, and the rate of the quantizer is $R = \log_2 M$.
- Decoder β: It is a one-to-one map $\mathcal{I} \to \mathcal{C}$, where $\mathcal{C}$ is the codebook composed of output levels y_i's, $\mathcal{C} = \{y_0, y_1, \dots, y_{M-1}\}$. So the output of the quantizer is $\hat{x} = Q(x) = \beta(\alpha(x)) = y_i$ if $i = \alpha(x)$.

A specific quantizer is the M-level (scalar) uniform one. Output levels of the decoder, y_i's ($i = 0, 1, \dots, M-1$), are equally spaced. Each y_i is the midpoint of the quantization interval (a_i, a_{i+1}), i.e., $y_i = (a_i + a_{i+1})/2$, where a_i's are thresholds. If the signal x is bounded in the range $[-L, L]$, or the granular region is $[-L, L]$ ($a_0 = -L$ and $a_M = L$), the step size should be $\Delta = 2L/M$. Consequently, output levels are

$$y_0 = -L + \Delta/2, \tag{1a}$$

$$y_i = y_0 + i\Delta, \qquad i = 0, 1, \dots, M-1. \tag{1b}$$

Since uniform quantizers are common, we first consider the effect of noise on the quantization performance of such a uniform quantizer Q. When the signal and noise are independent and the pdfs of them are $p_x(\cdot)$ and $p_n(\cdot)$ respectively, the MSE distortion can be expressed as

$$\begin{aligned} D &= E\left[(\hat{x} - x)^2\right] \\ &= E\left[(Q(x+n) - x)^2\right] \\ &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} p_x(x) p_n(n) (Q(x+n) - x)^2 \, dx dn. \end{aligned} \tag{2}$$

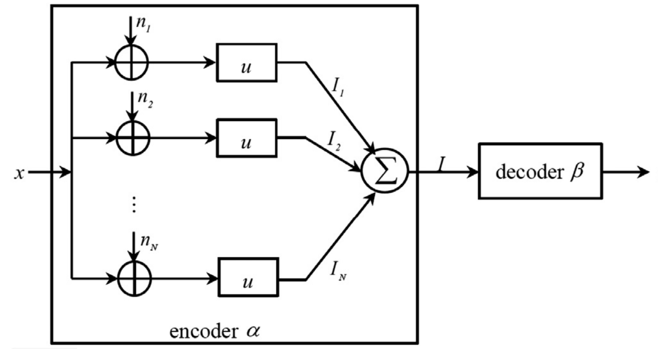

Fig. 1. A stochastic quantizer based on an array of binary quantizers (α) and a decoder (β).

To minimize the distortion D in Eq. (2), an optimization problem, $\min_{p_n(\cdot)} D$, should be examined. In Appendix A, we show that the solution to such a problem is $p_n(n) = \delta(n)$, where $\delta(\cdot)$ is the Dirac delta function. That is to say, random noise dose not enhance quantization performance, measured by MSE distortion, in the uniform quantization scenario (although the statistics of quantization error may benefit from the dithers [8]).

In some sense, adding noise to a deterministic uniform quantizer is equivalent to dithering thresholds in the same way. Therefore, MSE distortion will not decrease either, when all thresholds are randomized by the same dither.

3. Optimal noise in an array of binary quantizers

As noise has a negative effect in the uniform quantizer,[1] we focus on another system, which is shown in Fig. 1, to discuss the noise effect on quantization performance. Such an encoder—an array of threshold components—has been widely used to exhibit phenomena of SR or SSR [14]. It is also of practical importance, since it can be used for analog to digital conversion [12]. In Ref. [18], that array, combined with a probability density transform, is extended to a novel stochastic analog to digital converter. The converter can achieve high accuracy, and it is robust in harsh conditions where some threshold components may fail to work. Moreover, a similar array is used in Ref. [17], but each threshold component in the array is a multi-level quantizer instead of a binary one.

In the array in Fig. 1, there are N binary quantizing elements. All elements are subject to the same signal x, and have the identical threshold u, as considered in Refs. [12,18]. The signal's pdf is $p_x(\cdot)$. Moreover, noises n_i ($i = 1, 2, \dots, N$) are mutually independent and have the identical pdf $p_n(\cdot)$.

Since each binary quantizer maps the input signal x into {0, 1}

$$I_i = \begin{cases} 0, & x + n_i \le u \\ 1, & x + n_i > u \end{cases},$$

the set of indices produced by the encoder in Fig. 1 will be $\mathcal{I} = \{0, 1, \dots, N\}$. That is to say, $\alpha(x, n_1, \dots, n_N) = \sum_{i=1}^{N} I_i = I \in \mathcal{I}$, the number of quantization levels is M and the rate is $R = \log_2 M$, where $M = N + 1$.

For the decoding operation, a common decoder used in uniform quantization scenario is considered here. Its output points y_i's ($i = 0, 1, \dots, M-1$) are equispaced as expressed in Eq. (1), and the step size is Δ. Such a decoding operation is also called linear decoding [14].

[1] Note that the performance of quantizers is measured by MSE distortion.