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Persymmetric adaptive detection of distributed targets in compound-Gaussian sea clutter with Gamma texture

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ABSTRACT

We consider the problem of detecting a distributed target in compound-Gaussian sea clutter with Gamma distributed texture components. We design the Rao, Wald, and generalized likelihood ratio test (GLRT) detectors according to the two-step method: the first step is to obtain the Rao, Wald tests and GLRT on the assumption that the texture or/and covariance matrix structure are known; then the maximum a posteriori probability estimate of the clutter texture and the fixed point estimate of covariance matrix exploiting persymmetry are employed to replace the known texture and covariance matrix in the tests derived in the first step. Remarkably these proposed detectors ensure constant false alarm rate with respect to the covariance matrix structure. The effectiveness of the proposed detectors is verified by using simulated and real sea clutter data.

1. Introduction

The problem of detecting distributed targets has received considerable attention in the literature [1–6]. Much work has been done for this problem in Gaussian clutter (see [7,8], and references therein). Nevertheless, the Gaussian model is not suitable for clutter in many practical scenarios, e.g., in sea clutter or high resolution radars. As an alternative, a compound-Gaussian model for rational physical interpretation mechanism, is widely used to describe the statistical properties of clutter [9–15]. In this model, clutter can be expressed as the product of two mutually independent components: texture and speckle. The texture component varies slowly, which describes the reflectivity of illuminated areas and is a non-negative real random process; the speckle component varies more rapidly, which accounts for the local backscattering and has a complex, zero-mean, possibly correlated Gaussian distribution due to the central limit theorem [16].

Recently, many studies have been done for distributed target detection in compound-Gaussian clutter. In [17–20], the generalized likelihood ratio test (GLRT), Rao and Wald tests were designed for the distributed target detection in the compound-Gaussian clutter with deterministic but unknown texture components. In [21], a subspace model was used for target returns, and a two-step adaptive subspace detector was designed for detecting distributed targets. The authors in [22] derived a GLRT for adaptively detecting range and Doppler-distributed targets in compound-Gaussian clutter whose texture is range dependent. The problem of detecting a distributed target in compound-Gaussian clutter in polarimetric multiple input multiple output radar was addressed in [23], where a two-step GLRT was proposed. In [24], the speckle component in the compound-Gaussian clutter was described by an autoregressive (AR) process, and an AR-based GLRT was developed for the distributed target detection.

It is known that compound-Gaussian distribution is referred to as *K*-distribution and *t*-distribution, when the texture component is subject to Gamma and inverse Gamma distributions, respectively. In the compound-Gaussian clutter with inverse Gamma texture, three adaptive detectors (one-step, a-posteriori, and two-step GLRTs) were derived in [25] for detecting distributed targets. In [26], the persymmetric Rao and Wald tests were developed for the problem of detecting a distributed target in compound-Gaussian clutter with unknown covariance matrix, where the texture component is assumed to follow an inverse-Gamma distribution. The choice of a Gamma distribution for texture is widely accepted and verified [27,28]. In compound-Gaussian clutter with Gamma distributed texture, the detection problem of a point-like (instead of distributed) target was investigated in [29–33]. Bandiera et al. developed a Bayesian GLRT detector without using secondary data for the distributed target detection problem in compound-Gaussian

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noise with Gamma distributed texture [34], where the covariance matrix structure has an inverse Wishart distribution.

In this work, we consider the problem of detecting a distributed target in compound-Gaussian clutter with Gamma distributed texture. Secondary data are assumed available for the estimation of covariance matrix structure. Our contributions are listed as follows:

1) Three detectors are derived in two steps on basis of the principles of the GLRT, Rao and Wald tests. Note that the Rao or Wald test used here is different from the conventional one. The conventional Rao or Wald test requires to estimate all the unknown parameters in one step, which is infeasible for the problem at hand. Alternatively, we design the Rao or Wald test in two steps. In the first step, the Rao or Wald test is designed by assuming that the texture and covariance matrix structure are known. Then the maximum a posterior (MAP) estimate of texture and the fixed point estimate of the covariance matrix structure are obtained to replace the texture and covariance matrix structure, respectively, in the Rao or Wald test derived in the first step.

2) Persymmetry is exploited in the covariance matrix to alleviate the requirement on the training data size. The persymmetry exists in clutter covariance matrix when a detection system is equipped with a symmetrically-spaced linear array or symmetrically-spaced pulse trains. In such cases, the persymmetric covariance matrix has a property of double symmetry, i.e., Hermitian about its principal diagonal and symmetric about its cross diagonal.

3) We give a proof that the proposed detectors exhibit constant false alarm rate (CFAR) property against the covariance matrix structure. Numerical results based on simulated and real data reveal that the proposed detectors outperform their counterparts.

The remainder of this paper is organized as follows. Section 2 presents problem formulation. In Section 3, the Rao test, Wald test and GLRT are derived. In Section 4, the CFAR property of the proposed detectors against the structure of covariance matrix is proved. The effectiveness of the proposed detectors is verified by using simulated and real data in Section 5. Finally the paper is summarized in Section 6.

Notation: Vectors (matrices) are denoted by boldface lower (upper) case letters. Superscripts $(\cdot)^T$, $(\cdot)^*$ and $(\cdot)^\dagger$ denote transpose, complex conjugate and complex conjugate transpose, respectively, The notation $\sim$ means "is distributed as," and $\mathcal{CN}$ denotes a circularly symmetric, complex Gaussian distribution, $\Gamma(\cdot)$ is the Gamma function, $K_\nu(\cdot)$ denotes the modified Bessel function of the second kind with order ν. Re$[\cdot]$ and Im$[\cdot]$ denote the real and imaginary parts of the argument, respectively. $E(\cdot)$ is the statistical expectation. diag$(\cdot)$ stands for a square diagonal matrix with the elements of a given vector on the diagonal, $|\cdot|$ represents the modulus of a complex number. Tr$(\cdot)$ denotes the trace of a matrix, $\jmath = \sqrt{-1}$, and det$(\cdot)$ denotes the determinant of a matrix.

2. Problem formulation

Consider a radar system with N channels receiving the signal echoes reflected from a distributed target, which (if present) occupies H successive range bins. The received data in the hth range bin of the distributed target is denoted by an $N \times 1$ column vector $\mathbf{z}_h, h = 1, \dots, H$. These data (called primary data) can be represented by

$$\mathbf{z}_h = \alpha_h \mathbf{v} + \mathbf{n}_h, \quad h = 1, \dots, H, \tag{1}$$

where $\mathbf{v}$ is the known signal steering vector, α_h is a deterministic but unknown complex scalar accounting for the target reflectivity and channel propagation effects, and $\mathbf{n}_h$ denotes sea clutter in the hth primary data. Assume that the sea clutter $\mathbf{n}_h$ has compound-Gaussian distribution, i.e.,

$$\mathbf{n}_h = \sqrt{\tau_h}\mathbf{g}_h, \quad h = 1, \dots, H, \tag{2}$$

where the variable τ_h, is a positive random variable, usually referred to as texture, representing the local clutter power; the column vector $\mathbf{g}_h$ is an N dimensional random vector, usually named speckle, representing the properties of the coherent radar channels.

The speckle component $\mathbf{g}_h$ is modeled as a circular complex Gaussian vector with zero mean and positive definite covariance matrix $E[\mathbf{g}_h\mathbf{g}_h^\dagger] = \mathbf{R}$. In shorthand notation, we write $\mathbf{g}_h \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$. When the used system is equipped with a symmetrically spaced linear array or symmetrically spaced pulse trains [35], the matrix $\mathbf{R}$ has a property of double symmetry, i.e.,

$$\mathbf{R} = \mathbf{R}^\dagger, \quad \text{and} \quad \mathbf{R} = \mathbf{J}\mathbf{R}^*\mathbf{J}, \tag{3}$$

where $\mathbf{J}$ is a permutation matrix with unit anti-diagonal elements and zeros elsewhere, namely,

$$\mathbf{J} = \begin{bmatrix} 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & \cdots & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & \cdots & 0 & 0 \\ 1 & 0 & \cdots & 0 & 0 \end{bmatrix}. \tag{4}$$

As for the distribution of the texture component, we consider the case that τ_h is subject to Gamma distribution, which is widely accepted and verified by the statistical analysis on measured data [27]. The probability density function(PDF) of τ_h is given by [36]

$$f(\tau_h) = \frac{\beta_h^{q_h}}{\Gamma(q_h)}\tau_h^{q_h-1}\exp(-\beta_h\tau_h), \quad \tau_h \ge 0, q_h \ge 0, \beta_h \ge 0 \tag{5}$$

where q_h and β_h are the shape and scale parameters, respectively.

Now we need to make a decision upon a binary hypothesis. That is, under the null hypothesis H_0 the received data $\mathbf{z}_h$ only contains the clutter $\mathbf{n}_h$. In contrast, under the alternative hypothesis H_1, $\mathbf{z}_h$ consists of the clutter $\mathbf{n}_h$ and target signal. In summary, the detection problem at hand can be formulated as the following binary hypothesis testing:

$$H_0 : \begin{cases} \mathbf{z}_h = \mathbf{n}_h, & h = 1, \dots, H, \\ \mathbf{z}_k = \mathbf{n}_k, & k = H+1, \dots, H+K, \end{cases} \tag{6a}$$

and

$$H_1 : \begin{cases} \mathbf{z}_h = \alpha_h\mathbf{v} + \mathbf{n}_h, & h = 1, \dots, H, \\ \mathbf{z}_k = \mathbf{n}_k, & k = H+1, \dots, H+K. \end{cases} \tag{6b}$$

Usually, the shape and scale parameters of the texture components in the clutter are unknown. In practice, we can use the method of moments to estimate these parameters [27,37]. Hence, we assume that the shape and scale parameters are known in the detection problem. The detection schemes designed with known shape and scale parameters can provide a benchmark of the best achievable performance.

Note that the positive definite covariance matrix structure $\mathbf{R}$ is unknown in practice. To estimate it, a set of training (secondary) data $\mathbf{z}_k, k = H+1, H+2, ..., H+K,$ only containing clutter is assumed available. They are usually collected in the vicinity of the primary data [38,39]. Suppose that the clutter in the training data has compound-Gaussian distribution, and shares the same covariance matrix structure $\mathbf{R}$ with the clutter in the primary data.

It is worth noticing that the GLRT detector has been proposed for the detection problem in the point-like target case (i.e., $H = 1$) [29]. However, no detector is designed in the distributed target case for the detection problem (6). In the following, we propose three adaptive detectors with $H > 1$ for (6).