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Short communication

Performance of soft limiters in the LMS algorithm for cyclostationary white Gaussian inputs☆

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ABSTRACT

The analysis of saturation-type nonlinearities on the input and the error in the weight update equation for LMS adaptation were obtained for a stationary white Gaussian data model in [28] for system identification. Here the input signal is modeled by a cyclostationary white Gaussian random process with periodically time-varying power. The system parameters vary according to a random-walk. Using the previous analysis results, nonlinear recursions are presented for the transient and steady-state weight first and second moments that include the effect of the soft limiters. Monte Carlo simulations of the algorithms provide strong support for the theory.

1. Introduction

An important aspect of adaptive filter performance is the ability to track time variations of the underlying signal statistics [1,2]. The standard analytical model assumes the input signal is stationary (see e.g. [3–11]). However, a non-stationary signal model can be provided by a random walk model for the optimum weights. The form of the mean-square error performance surface remains unaltered while the surface moves in the weight space over time. This model provides the conditions for the adaptive algorithm to track the optimum solution [1]. Alternatively, the input signal can be modeled as a cyclostationary process in many practical applications [12–14]. In these cases, the form of the performance surface is periodic with the same period as the input autocorrelation matrix [15]. This performance surface deformation affects the adaptive filter convergence and is independent of changes in the optimum weights. This transient performance surface deformation can be modeled by standard analytical models.

However, it is still desirable to understand the adaptive performance with non-stationary inputs. In particular, adaptive solutions have been sought for many application areas involving cyclostationary signals [16].

The history of the analysis of the stochastic behavior of adaptive algorithms for nonstationary inputs is relatively limited. LMS behavior for cyclostationary inputs was first studied in [17] but only for convergence of the mean weights. The special case of pulsed input power variations was studied in [18,19] for both LMS and NLMS algorithms with linear combiner structures. The stochastic behavior of the LMS and NLMS algorithms for two types of cyclostationary white Gaussian inputs was studied in [20,21]. The behavior of the Least Mean Fourth (LMF) algorithm with nonstationary inputs has been investigated in [22]. Bershad et al. [23] studied the second moment behavior of the adaptive line enhancer/adaptive line canceler for a cyclostationary input which consisted of a fixed amplitude random phase sine wave plus a white Gaussian process with periodic power variations. The above analyses were extended to signed LMS algorithms [24]. Adaptive filtering based on the time averaged MSE has been applied to cyclostationary signals [25]. None of these works considered the application of nonlinearities to the error or to the input regressor in the weight update. Non-linear effects for the LMS algorithm have been studied for stationary inputs [26]. Recently the LMS and NLMS algorithms were compared for cyclostationary inputs [27]. Most recently, a unified theory for the LMS algorithm with soft limiters [28] has been developed for stationary inputs. This paper uses the theory in [28] to extend to cyclostationary inputs.

Adaptive filter analysis for cyclostationary inputs is not easy because of the difficulty of modeling the input cyclostationarity in a mathematically treatable way. This point is very important for soft limited LMS algorithms because of the algorithm non-linearities. Thus, relatively simple models are needed to infer algorithm behavior for inputs with time-varying statistics.

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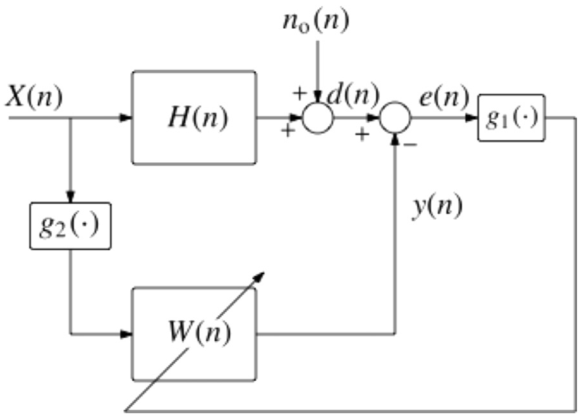

Fig. 1. Adaptive plant identification.

This note extends previous statistical analyses of the soft limited LMS algorithms [28] with stationary white Gaussian input signals in a system identification framework to the cyclostationary case. The input cyclostationary signal is modeled by a white Gaussian random process with periodically time-varying power. These models are used to study the adaptive filter performance for input signals with fast, moderate or slow sinusoidal and pulsed power variations. Simulation results show excellent agreement with the theoretically predicted behaviors, confirming the usefulness of the analytical model to study the adaptive filter behavior.

The note is organized as follows. Section 2 defines the problem and the statistical assumptions used to solve the problem. Section 3 extends the general recursions for the mean and second moment weight behavior for the soft limited LMS algorithm [28] for the stationary case to the cyclostationary case. These recursions are studied for soft limiting of both the input and error of the LMS algorithm for identifying a Markov channel with the two cyclostationary inputs. Section 4 compares the developed theory with Monte Carlo simulations. Section 5 presents the conclusions. Capital letters denote vectors or matrices, and small letters denote scalar variables.

2. Problem definition and statistical assumptions

2.1. System identification and the Markov channel model

This paper will study the system identification model given in Fig. 1. All signals and systems are real. The *N*-dimensional input vector to the adaptive filter tap weights is given by $X(n) = [x(n),\ x(n-1),\ldots\ldots x(n-N+1)]^T$. The superscript T means transpose. The observation noise $n_o(n)$ is assumed stationary zero-mean white Gaussian with variance σ_o^2 and independent of *X(n)*. The standard random walk model [1,2] is used for the unknown channel

$$H(n+1) = H(n) + Q(n) \tag{1}$$

where *Q(n)* is a white Gaussian vector with zero mean and covariance matrix $E[Q(n)Q^T(n)] = \sigma_q^2\ I$ where I is the identity matrix. The vector sequence *Q(n)* is assumed independent of both *X(n)* and $n_o(n)$. This model is the so-called random walk approximation to the first order Markov model [28]. The random walk model (1) is not realistic. However, it allows a feasible tracking analysis that provides important insights into the ability of the adaptive algorithm to track channel variations [26].

The well-known adaptive filter Independence Theory (IT) assumes that the adaptive filter weights at time *n*, *W(n)*, are statistically independent of the input vector *X(n)* [28]. The use of this assumption considerably simplifies the stochastic analysis of the adaptive filter. The IT assumption has been shown to lead to very accurate models in a wide variety of adaptive filter applications. The agreement between the theoretical results and Monte Carlo simulations in Section 5 supports the application of the IT assumption for cyclostationary input signals.

Define the weight deviation vector $V(n) = W(n) - H(n)$ and the weight deviation covariance matrix $K_{VV}(\mathrm{n}) = E[V(n)V^T(n)]$. Then the mean square deviation (*MSD*) is given by [28]

$$MSD(\mathrm{n}) = E\left[V^T(n)V(n)\right] = trace[K_{VV}(n)] \tag{2}$$

where $trace[B]$ is the trace of the matrix B. The IT assumption is needed when evaluating the recursions for the mean weight and $K_{VV}(\mathrm{n})$ as will be shown shortly.

2.2. Nonlinear LMS algorithm

The conventional LMS adaptive algorithm is given by

$$W(n+1) = W(n) + \ \mu e(n)X(n) \tag{3}$$

where

$$e(n) = \ H^T(n)X(n) + n_o(n) - W^T(n)X(n) \tag{4}$$

and μ is the step-size.

The nonlinear LMS adaptive algorithm studied here is given by

$$W(n+1) = W(n) + \mu g_1[e(n)]G_2[X(n)] \tag{5}$$

where $G_2^T[X(n)] = [g_2[x(n)], g_2[x(n-1)], \ldots,\ g_2[x(n-N+1)]]$ and $g_1[.]$ and $g_2[.]$ are bounded odd nonlinearities.

2.3. Cyclostationary input signal model

A wide sense cyclostationary random process $y(t)$ is defined [1-p.82] as

$$E[y(t_1+T)] = E[y(t_1)],\ \ E[\ y(t_1+T)y(t_2+T)] = E[y(t_1)y(t_2)] \tag{6}$$

for all t_1 and t_2 and where T is the period.

It is assumed that the elements of the input vector *X(n)*, $x(n-k),\ k = 0,\ldots,\ N-1$ are samples of a zero-mean white Gaussian sequence with time-varying variance. Thus, the autocorrelation matrix $R(n)$ is given by

$$\begin{aligned} R(n) &= E\left[X(n)X^T(n)\right] \\ &= Diag\left[\sigma_x^2(n), \sigma_x^2(n-1)\ ,\ldots,\ \sigma_x^2(n-N+1)\right] \end{aligned} \tag{7}$$

where $\sigma_x^2(n)$ is periodic with period T. Hence, *X(n)* is a discrete time wide sense cyclostationary process. Although this model is not general, Eq. (7) defines a non-trivial model. It allows the input to display a simple type of cyclostationarity which can be used to model more complex time-varying statistical properties of the inputs. Two simple models for $\sigma_x^2(n)$ are considered here: a sinusoidal power time variation

$$\sigma_x^2(n) = \beta(1+\sin(\omega_o n)) \tag{8}$$

and a pulsed power time variation

$$\begin{aligned} &\sigma_x^2(n) = P_1\ for\ iT < n \le iT + \alpha T, \\ &\sigma_x^2(n) = P_2\ for\ iT + \alpha T < n \le (i+1)T, \\ &\qquad \text{for } 0 < \alpha < 1 \quad \text{and} \quad i\ = 1, 2, \ldots \end{aligned} \tag{9}$$

The theory presented here can be extended to other cyclostationary power variations in a similar manner.

The sinusoidal variation model can be used to study the algorithm behavior for different speeds of input power variation with bounded maximum power. The pulsed model can be used to study the algorithm behavior for pulsed inputs such as those occurring in