Automatica 50 (2014) 2077–2083

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Observability of linear systems with commensurate delays and unknown inputs☆

Francisco Javier Bejarano[a,1], Gang Zheng[b]

[a] SEPI, ESIME Ticomán, IPN, Av. San José Ticomán 600, C.P. 07340, Mexico City, Mexico
[b] Non-A, INRIA - Lille Nord Europe, 40 avenue Halley, Villeneuve d'Ascq 59650, France

ARTICLE INFO

Article history:
Received 7 July 2013
Received in revised form
27 January 2014
Accepted 10 April 2014
Available online 20 June 2014

Keywords:
Delay systems
Commensurate delays
Observability
Unknown inputs

ABSTRACT

This paper investigates the observability analysis for linear time systems whose outputs are affected by unknown inputs. Three different definitions of observability are proposed. By introducing the Smith form and comparing the invariant factors, a sufficient condition is deduced for each proposed observability definition. Three examples are given for the purpose of highlighting the effectiveness of the proposed approach.

1. Introduction

Time delay systems are widely used to model many concrete applications, like chemical and biological processes and many results have been published to treat these kinds of systems for different aspects (Richard (2003) and Sename (2001)). The analysis of observation for time delay systems can be dated back to the 1980s (Lee and Olbrot (1981), Olbrot (1981), Rabah (1995) and Salamon (1980)). For this issue, different definitions of observability have been proposed, such as strong observability, spectral observability and weak observability.

For linear time delay systems, various aspects of the observability problem have been studied in the literature, using different methods such as the functional analytic approach (Bhat & Koivo, 1976) or the algebraic approach (Brewer, Bunce, and Vleck (1986), Fliess and Mounier (1998) and Sontag (1976)). For non-linear time delay systems, by using the theory of non-commutative rings (Moog, Castro-Linares, Velasco-Villa, & Marque-Martinez, 2000), the observability problem has been studied in Xia, Marquez, Zagalak, and Moog (2002) for systems with known inputs, and in Zheng, Barbot, Boutat, Floquet, and Richard (2011) for systems with unknown inputs. The associated observer for some classes of time delay systems can be found in Conte, Perdon, and Guidone-Peroli (2003), Darouach (2006), Fattouh, Sename, and Dion (1999) Fu, Duan, and Song (2004) and Sename (2001) and the references therein.

Nonetheless, the majority of the existing works on observability analysis are focused on time delay systems whose outputs are not affected by unknown inputs. However, this situation might exist in some practical applications and this motivates the work of this paper. Here, we deal with time delay systems which are linear and whose delays are commensurable. We consider that delays may appear in the state, input, and output. The aim is searching for some conditions allowing for the reconstruction of the entire state vector using backward, actual, and\or forward output information.

The contributions of this paper are as follows. First, we introduce the Unknown Input Observability (UIO), backward UIO and forward UIO concepts. For each one of the proposed observability definitions, we obtain sufficient conditions that can be verified by using some matrices depending on the original system parameters. The established condition for the unknown input observability turns out to be a generalization of the already known condition for systems with unknown inputs, but without delays (in that case such a condition is also a necessary one), and also it is a generalization of the known strongly observable condition for linear systems with commensurable delays, but without unknown inputs. Due to

☆ The first author acknowledges the support of IPN SIP project 20141364. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Akira Kojima under the direction of Editor Ian R. Petersen.

E-mail addresses: javbejarano@yahoo.com.mx (F.J. Bejarano), gang.zheng@inria.fr (G. Zheng).

1 Tel.: +52 55 5729 6000x4613; fax: +52 55 5729 6000.

http://dx.doi.org/10.1016/j.automatica.2014.05.032
0005-1098/

the methodology used in the paper, the results may be applied to systems over polynomial rings.

The following notations will be used: $\mathbb{R}$ is the field of real numbers, $\mathbb{R}_{\neq 0}$ is the set of nonzero real numbers. The set of nonnegative integers is denoted by $\mathbb{N}_0$. $\mathbb{R}[\delta]$ is the polynomial ring over the field $\mathbb{R}$. The Laurent polynomial ring is denoted as $\mathbb{R}\left[\delta, \delta^{-1}\right]$. $\mathbb{R}^n[\delta]$ is the $\mathbb{R}[\delta]$-module whose elements are the vectors of dimension n and whose entries are polynomials. By $\mathbb{R}^{q\times s}[\delta]$ we denote the set of matrices of dimension $q \times s$, whose entries are in $\mathbb{R}[\delta]$. For $f(\delta)$, a polynomial of $\mathbb{R}[\delta]$, $\deg f(\delta)$ is the degree of $f(\delta)$. For a matrix $M(\delta)$, $\deg M(\delta)$ (the degree of $M(\delta)$) is defined as the maximum degree of all the entries $m_{ij}(\delta)$ of $M(\delta)$. $\det M(\delta)$ is the determinant of this matrix, and $\operatorname{rank} M(\delta)$ means the rank of the matrix $M(\delta)$ over $\mathbb{R}[\delta]$. The acronym for greatest common divisor is *gcd*.

2. Formulation of the problem and definitions

We will deal with the following class of linear systems with commensurate delays:

$$\begin{aligned} \dot{x}(t) &= \sum_{i=0}^{k_a} A_i x(t-ih) + \sum_{i=0}^{k_b} B_i w(t-ih) \\ y(t) &= \sum_{i=0}^{k_c} C_i x(t-ih) + \sum_{i=0}^{k_d} D_i w(t-ih) \end{aligned} \tag{1}$$

where the state vector $x(t) \in \mathbb{R}^n$, the system output vector $y(t) \in \mathbb{R}^p$, and the unknown input vector $w(t) \in \mathbb{R}^m$, the initial condition $\varphi(t)$ is a piecewise continuous function $\varphi(t) : [-kh, 0] \to \mathbb{R}^n$ ($k = \max\{k_a, k_b, k_c, k_d\}$); thereby $x(t) = \varphi(t)$ on $[-kh, 0]$. A_i, B_i, C_i, and D_i are matrices of appropriate dimension with entries in $\mathbb{R}$. By using the delay operator (backward time-shift operator) $\delta : x(t) \to x(t-h)$, system (1) may be represented in the following compact form:

$$\begin{aligned} \dot{x}(t) &= A(\delta)x(t) + B(\delta)w(t) \\ y(t) &= C(\delta)x(t) + D(\delta)w(t) \end{aligned} \tag{2}$$

where $A(\delta)$, $B(\delta)$, $C(\delta)$, and $D(\delta)$ are matrices over the polynomial ring $\mathbb{R}[\delta]$, defined as $A(\delta) := \sum_{i=0}^{k_a} A_i\delta^i$, $B(\delta) := \sum_{i=0}^{k_b} B_i\delta^i$, $C(\delta) := \sum_{i=0}^{k_c} C_i\delta^i$, and $D(\delta) := \sum_{i=0}^{k_d} D_i\delta^i$. As for $x(t; \varphi, w)$, we mean the solution of the delay differential equation of system (1) with the initial condition equal to φ, and the input vector equal to w. Analogously, we define $y(t; \varphi, w) := C(\delta)x(t; \varphi, w) + D(\delta)w(t)$, that is, to be the system output of (1) when $x(t) = x(t; \varphi, w)$.

Practically, what we search for is to find out conditions allowing for the estimation of $x(t)$. To tackle the problem in a more formal way, we use the following observability definitions.

Definition 1 (*Unknown Input Observability*)**.** System (1) is called *unknown input observable* (*UIO*) on the interval $[t_1, t_2]$ iff there exist t_1' and t_2' ($t_1' < t_2'$) such that, for all inputs w and every initial condition φ,

$$y(t; \varphi, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right]$$
$$\text{implies } x(t; \varphi, w) = 0 \text{ for } t \in [t_1, t_2].$$

Definition 2 (*Backward UIO*)**.** System (1) is said to be *backward UIO* (*BUIO*) on $[t_1, t_2]$ iff for each $\tau \in [t_1, t_2]$ there exist $t_1' < t_2' \le \tau$ such that, for all inputs w and every initial condition φ,

$$y(t; \varphi, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right] \text{ implies } x(\tau; \varphi, w) = 0.$$

Definition 3 (*Forward UIO*)**.** System (1) is said to be *forward UIO* (*FUIO*) on $[t_1, t_2]$ iff for each $\tau \in [t_1, t_2]$ there exist $t_2' > t_1' \ge \tau$ such that, for all inputs w and every initial condition φ,

$$y(t; \varphi, w) = 0 \quad \text{for all } t \in \left[t_1', t_2'\right] \text{ implies } x(\tau; \varphi, w) = 0.$$

Remark 1. These definitions are essentially formulated following the observability definitions given in Kalman, Falb, and Arbib (1969) for linear systems. Basically, UIO considers the case when the state vector can be reconstructed using past, actual, and future values of the system output. As for BUIO, it is related with the case when only actual and past values of the system output are needed for the actual state reconstruction. Finally, FUIO defines a property which theoretically allows for the reconstruction of the actual state vector using only actual and future values of the system output.

Obviously, either BUIO or FUIO implies UIO. It should be noted that BUIO and FUIO do not exclude each other. For instance, the system

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = x_1 + \delta x_2; \qquad y_1 = \delta x_1, \qquad y_2 = x_2$$

is BUIO on $[t_1, t_1 + h]$ ($t_1 \ge h$) since, for each $\tau \in [t_1, t_1 + h]$, $y(t) = 0$ on $[t_1 - h, \tau]$ implies $x(\tau) = 0$. Moreover, it is FUIO on $[t_1, t_1 + h]$, since, $y(t) = 0$ on $[\tau, t_1 + 2h]$ implies $x(\tau) = 0$.

In the next section we will search for sufficient conditions allowing for the test of the UIO property, which at the same time provide us with a constructive way to reconstruct $x(t)$ in finite time.

3. Basic results

The study of the observability for linear systems (without delays) has been successfully tackled by using geometric methods, in particular invariant subspaces. For the time delay case such methods cannot be followed straightforwardly, but still many of those ideas can be borrowed (see Conte et al. (2003) and Conte, Perdon, and Moog (2007)). Here we will not follow strictly a geometric method; however, in its spirit the idea still comes from the results of geometric methods of standard linear systems, as we will see below.

Let $P(\delta)$ be a matrix of $q \times s$ dimension with rank equal to r (clearly $r \le \min\{q, s\}$). We know that there exists an invertible matrix $T(\delta)$ over $\mathbb{R}[\delta]$ (representing elementary row operations) such that $P(\delta)$ is put into (column) Hermite form. Thus, we have that

$$T(\delta)P(\delta) = \begin{bmatrix} P_1(\delta) \\ 0 \end{bmatrix}$$

where $P_1(\delta)$ is of $r \times s$ dimension, and $\operatorname{rank} P_1 = r$. Also, there exist two invertible matrices $U(\delta)$ and $V(\delta)$ over $\mathbb{R}[\delta]$ (representing elementary row and column operations, respectively) such that $P(\delta)$ is reduced to its Smith form, i.e.,

$$U(\delta)P(\delta)V(\delta) = \begin{bmatrix} \operatorname{diag}(\psi_1(\delta)\cdots\psi_r(\delta)) & 0 \\ 0 & 0 \end{bmatrix}$$

where the $\{\psi_i(\delta)\}$'s are *monic nonzero* polynomials satisfying

$$\psi_i(\delta) \,|\, \psi_{i+1}(\delta) \quad \text{and} \quad d_i(\delta) = d_{i-1}(\delta)\,\psi_i(\delta)$$

where $d_i(\delta)$ is the gcd of all $i \times i$ minors of $P(\delta)$ ($d_0 = 1$). The $\{\psi_i(\delta)\}$'s are called invariant factors, and the $\{d_i(\delta)\}$'s determinant divisors.

Following the ideas of Molinari (1976) and Silverman (1969), let us define $\{\Delta_k(\delta)\}$ matrices generated by the following algorithm,

$$\begin{aligned} &\Delta_0 \triangleq 0, \qquad G_0(\delta) \triangleq C(\delta), \qquad F_0(\delta) \triangleq D(\delta) \\ &S_k(\delta) \triangleq \begin{bmatrix} \Delta_k(\delta)B(\delta) \\ F_k(\delta) \end{bmatrix}, \quad k \ge 0 \\ &\begin{bmatrix} F_{k+1}(\delta) & G_{k+1}(\delta) \\ 0 & \Delta_{k+1}(\delta) \end{bmatrix} \triangleq T_k(\delta) \begin{bmatrix} \Delta_k(\delta)B(\delta) & \Delta_k(\delta)A(\delta) \\ F_k(\delta) & G_k(\delta) \end{bmatrix} \end{aligned} \tag{3}$$

where $T_k(\delta)$ is an invertible matrix over $\mathbb{R}[\delta]$ that transforms S_k into its Hermite form, and $\Delta_0$is of dimension 1 by n. Then, $\{M_k(\delta)\}$