Automatica 50 (2014) 2084–2089

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Near-minimum-time control of asymmetric rigid spacecraft using two controls[☆]

Donghoon Kim[1], James D. Turner

Department of Aerospace Engineering, Texas A&M University, College Station, TX 77843-3141, USA

ARTICLE INFO

Article history:
Received 18 January 2013
Received in revised form
11 November 2013
Accepted 11 April 2014
Available online 20 June 2014

Keywords:
Euler-angle transformation
Minimum time
Asymmetric
Sequential maneuver
Switch times
Final time

ABSTRACT

In general, spacecraft are designed to be maneuvered to achieve pointing objectives. To reorient the spacecraft with zero angular velocity at the end of the maneuver, a three-axis control design is usually used. When an actuator fails among three actuators, one must achieve these objectives using two control inputs, so that new control laws need to be considered. A simple and novel control law, which is based on the sequential Euler angle rotation strategy, is addressed. This paper explores a near minimum time control problem with constrained control input magnitudes. By introducing the three sequential sub-maneuvers with Euler-angle transformations, the governing nonlinear equations become rigorously linear, which permits a closed-form solution to be obtained for the switch times and final time, where the three sub-maneuvers are coupled through the unknown switch times. A numerical example demonstrates that the three-dimensional maneuver for an asymmetric spacecraft with two constrained control inputs can be successfully performed using the proposed closed-form solution.

Published by Elsevier Ltd.

1. Introduction

This work addresses the problem of reconfiguring spacecraft maneuvering control laws for handling an under-actuated system (which models an actuator failure), where only two functioning actuators along the unique body axes are assumed to be available. Many researchers have proposed successful algorithms for controlling the attitude motion of rigid and flexible spacecraft when three control actuators are available (Junkins & Turner, 1980, 1986; Turner & Junkins, 1980). Numerous feedback control formulations are available for handling off-nominal control strategies. For example, Tsiotras and Longuski (1995) considered the problem of designing control strategies for handling situations where sensor and actuator failures limit the control options available for carrying out the original mission objectives. Keraï (1995) examined the more extreme case where only a single control actuator is available; not surprisingly, he concluded that the single actuator case is not controllable. Brockett (1983) showed that the two actuator control cases can be made asymptotically stable about the origin. Shen and Tsiotras (1999); Tsiotras (1997); Tsiotras, Corless, and Longuski (1995); Tsiotras and Doumtchenko (2000); Tsiotras and Luo (1997, 1998, 2000); Tsiotras and Schleicher (2000) further addressed the problem of stabilization of axis-symmetric spacecraft by developing a tracking control law formulation. Coron and Keraï (1996) and Morin, Samson, Pomet, and Jiang (1995) presented approximate strategies that switch between two different control laws. Kim, Turner, and Junkins (2014a,b) suggested a general optimal control solution minimizing control torque using the Homotopy approach, but it is computationally expensive. Krishnan (1992) introduced a sequential maneuver strategy for three-dimensional (3-D) reorientation of spacecraft, but more torque consumption is required by performing unnecessary sub-maneuvers. Recently, Kim (2010); Kim, Turner, and Leeghim (in press) suggested a simple and novel way to handle the failure control problem by introducing a sequential maneuver approach with Euler-angle transformations, which provides two possible sets of three successive rotations. It avoids exciting nonlinear coupling interaction effects in the equation of motion and attitude kinematics during sub-maneuvers (Kim & Turner, 2012, 2013a). Analytically, because the sub-maneuver problems are coupled by unknown switch times, the problem leads to a high dimensioned optimization problem, where it is very important to specify accurate starting guesses for the costates (Kim & Turner, 2013b). The problem is computationally challenging, because the numerical algorithm must deal with jump boundary conditions for the unknown switch times that must be iteratively solved (Kim, Turner, & Leeghim, 2013).

[☆] The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Marco Lovera under the direction of Editor Toshiharu Sugie.

E-mail addresses: aerospace38@gmail.com (D. Kim), jdturner@tamu.edu (J.D. Turner).

[1] Tel.: +1 979 777 3758; fax: +1 979 845 6051.

http://dx.doi.org/10.1016/j.automatica.2014.05.038
0005-1098/Published by Elsevier Ltd.

In this work, the authors address the optimal near minimum time reorientation problem for an asymmetric rigid spacecraft with only two available constrained control inputs. The goal of the problem is to carry out three sequential single-axis maneuvers of unknown duration which are coupled through the appearance of unknown switch times in the necessary conditions for the optimal control solutions. A sub-optimal solution is obtained by coupling the necessary conditions for the sub-maneuvers. The solution is assumed to consist of three maneuver periods and five switch times: (1) one switch time for each maneuver (we have out of three sub-maneuvers) from bang–bang type controls and (2) two switch times from three sequential maneuvers. The attitude motion for the spacecraft is modeled using Euler angle kinematics because it has no singular point for the assumed sequential control with zero initial angular velocity.

The major contribution of this work includes developing a solvable set of necessary conditions for three coupled sub-maneuvers that yields a global capability for carrying out arbitrary 3-D spacecraft maneuvers that optimally handle the special case of a failed actuator. Three steps are required for completing the algorithm: (1) obtaining optimality conditions for the constrained near minimum time control problem, (2) developing analytic closed-form solutions for the switch times and final time, and (3) determining an optimal sequential Euler angle rotation sequence by comparing the numerical values obtained for each maneuver sequences' cost function. The proposed approach is demonstrated to render the nonlinear 3-D maneuver problem tractable by using on-board spacecraft computers.

2. Problem formulation

2.1. Dynamics and kinematics

The two actuator case considered herein models a failed actuator case for an on-orbit spacecraft. Our goal is to develop optimal near minimum time maneuver strategies that achieve the originally desired 3-D maneuver. As a concrete example, we assume that the control actuator operating about the third body axis has failed. The rotational dynamics equation of motion for an asymmetric rigid spacecraft controlled by the remaining two control inputs, along the first and second body axes, is given by Kim (2013)

$$\begin{Bmatrix} J_1\dot{\omega}_1 \\ J_2\dot{\omega}_2 \\ J_3\dot{\omega}_3 \end{Bmatrix} = \begin{Bmatrix} (J_2 - J_3)\,\omega_2\omega_3 \\ (J_3 - J_1)\,\omega_3\omega_1 \\ (J_1 - J_2)\,\omega_1\omega_2 \end{Bmatrix} + \begin{Bmatrix} u_1 \\ u_2 \\ 0 \end{Bmatrix} \tag{1}$$

where J_1, J_2, and J_3 are the principal moments of inertia; ω_1, ω_2, and ω_3 are the angular velocities along the body axes; and u_1 and u_2 are the control inputs along the body axes.

There are many potential sets of variables that can be used to describe the attitude motion of rotating spacecraft. Examples include: Euler angles, quaternions, direction cosine matrix (DCM), modified Rodrigues parameters (MRPs), etc. A common problem to be dealt with in choosing a set of attitude variables is the potential for encountering a geometric singularity that disrupts the numerical integration process. For example, the so-called gimbal lock singularity of Euler angles limits their utility for arbitrary large angle concepts. However, under the assumption that the initial angular velocity is zero, there is no singularity issue for Euler angles when sequential maneuvers are utilized, and this observation is exploited in the developments that follow. The Euler angle kinematic differential equation in terms of the angular velocities of the rigid spacecraft is given by Crassidis and Junkins (2012); Schaub and Junkins (2009)

$$\begin{Bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix} \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix} \tag{2}$$

where $[C]$ is the DCM, which is composed with the elements C_{ij} ($i, j = 1, 2$, and 3).

2.2. Single-axis special case

For a single-axis special case with zero initial angular velocities, the DCM in Eq. (2) reduces to $I_{3\times3}$ and angular velocities, which are not associated with the rotation axis, are zero. For example, the rotational dynamics for the 1-axis maneuver can be written as

$$\left\{\dot{\theta}_1,\ \dot{\theta}_2,\ \dot{\theta}_3\right\}^{\mathrm{T}} = \left\{\omega_1,\ 0,\ 0\right\}^{\mathrm{T}}. \tag{3}$$

Then, the kinematic and rotational dynamic equations for an arbitrary single-axis maneuver can be simply expressed as

$$\begin{Bmatrix} \dot{\theta} \\ \dot{\omega} \end{Bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{Bmatrix} \theta \\ \omega \end{Bmatrix} + \begin{Bmatrix} 0 \\ 1/J \end{Bmatrix} u \tag{4}$$

where $\{\theta,\ \omega\}^{\mathrm{T}}$ is the state vector and the control input is constrained by

$$|u(t)| \le u_{\max}. \tag{5}$$

The objective is to determine a control input to bring any given initial state to a desired final state, which is assumed to be the origin given by

$$\left\{\theta(T),\ \omega(T)\right\}^{\mathrm{T}} = \mathbf{0}_{2\times1} \tag{6}$$

where T is the *free* final time.

A minimum time control solution is developed by defining the Lagrange form of a performance index given by

$$\mathcal{J} = \int_{t_0}^{T} 1\,\mathrm{d}t \tag{7}$$

where t_0 is the *fixed* initial time. An equation of this type is defined for each sub-maneuver. As a result, a final maneuver time is recovered for each sub-maneuver.

3. Closed-form solution derivation

3.1. Optimality conditions

Using standard calculus of variations techniques, the Hamiltonian for the given problem is defined as

$$\mathcal{H} = 1 + \lambda_\theta\dot{\theta} + \lambda_\omega\dot{\omega} = 1 + \lambda_\theta\omega + \lambda_\omega\frac{u}{J} \tag{8}$$

where $\{\lambda_\theta,\ \lambda_\omega\}^{\mathrm{T}}$ is the costate vector, and one obtains the costate equations

$$\begin{Bmatrix} \dot{\lambda}_\theta \\ \dot{\lambda}_\omega \end{Bmatrix} = -\begin{Bmatrix} \dfrac{\partial\mathcal{H}}{\partial\theta} \\ \dfrac{\partial\mathcal{H}}{\partial\omega} \end{Bmatrix} = \begin{Bmatrix} 0 \\ -\lambda_\theta \end{Bmatrix}. \tag{9}$$

Because the final time is unspecified, the final time transversality condition is defined by

$$0 = \mathcal{H}(T) = 1 + \lambda_\theta(T)\omega(T) + \lambda_\omega(T)\frac{u(T)}{J}. \tag{10}$$