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Short communication

Adaptive estimation of Haar wavelet transform parameters applied to fuzzy prediction of network traffic

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ABSTRACT

In this paper, we propose an adaptive approach to estimate the energies of the wavelet and scale coefficients of the Haar wavelet transform used in the multifractal modeling of network traffic traces. Simulation results confirm that the estimates obtained for the modeling parameters in the wavelet domain are precise. In addition, we propose an equation to calculate the autocorrelation function of the underlying multifractal model in terms of these wavelet domain parameters. In order to enhance the prediction performance of network traffic traces, the autocorrelation function is used to update orthonormal basis functions in a fuzzy system. To validate the adaptive fuzzy prediction approach, simulations with real network traffic traces are carried out, showing that the proposed algorithm provides lower mean square errors than other algorithms in the literature.

1. Introduction

In the last decades, several studies have shown the importance of signal analysis using the wavelet transform due to its multiscale representation of signals [1–3].

One of the applications of wavelet transform is in network traffic modeling in order to describe behaviors such as long-range dependence and burst incidences at different time scales [4,5]. These characteristics may degrade network performance in relation to Gaussian and short-range dependence traffic flows [3,6].

The main multifractal models are based on multiplicative cascades, which are structures where an interval is divided randomly by multipliers, conserving the interval mass [3]. Thus, at the end of the division process, a correlated sequence is obtained, representing the network traffic samples. As examples of wavelet domain based multifractal models, we can cite: the Lognormal Beta [7] model and the MWM (Multifractal Wavelet Model) [3].

The MWM model consists of a multiplicative cascade in the Haar wavelet domain [8], where multiplicative cascade multipliers are computed based on the signal energy decay. Although the MWM model being suitable for modeling network traffic, it requires the application of the wavelet transform to the whole traffic trace or to all samples in a time window that is intending to apply the model. In other words, in its original formulation, the MWM does not update its parameters at each time instant that a traffic sample is provided.

In order to achieve high utilization of resources in communication networks and for better decision making, traffic prediction must be as accurate as possible. Fuzzy modeling is capable of precisely representing a nonlinear complex processes such as network traffic traces through the combination of linear local models [9]. Moreover, adaptive prediction algorithms are the most appropriate for real time multimedia applications due to on-line processing capability. Taking these informations into account, we propose an adaptive fuzzy prediction algorithm that incorporates a wavelet domain modeling of network traffic.

More specifically, in this paper we propose equations that allow us to compose an algorithm to adaptively estimate the energy decay of wavelet coefficients for network traffic modeling. Also, we propose an equation for the autocorrelation function of the wavelet domain based traffic model in order to obtain orthonormal basis functions for a TSK (Takagi-Sugeno-Kang) adaptive fuzzy system to predict network traffic samples.

2. Adaptive wavelet domain multifractal modeling

The Multifractal Wavelet Model (MWM) is based on a multiplicative cascade in the wavelet domain for network traffic modeling. The βMWM is a variation of the MWM where the beta distribution is used as the probability density of the cascade multipliers [3].

The MWM modeling process consists of computing the discrete Haar wavelet transform [8] for a fixed number of scales J of the

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traffic trace [3]. From the Haar wavelet transform, the wavelet $W_{j,i}$ and scale $U_{j,i}$ coefficients are obtained for each scale j, where $0 \leq j \leq J-1$.

According to [3], the wavelet coefficients are generated by $W_{j,i} = U_{j,i} A_{j,i}$, where $A_{j,i}$ is a random variable with the symmetric beta distribution $\beta(p_j, p_j)$ and p_j is the parameter that determines the beta distribution shape. The multipliers $A_{j,i}$ are selected in order to control the energy decay of the wavelet coefficients $W_{j-1,i}$, by the parameters p_j given as follows [3]:

$$\eta_j = \frac{E(W_{j-1,i}^2)}{E(W_{j,i}^2)} = \frac{2p_j + 1}{p_{j-1} + 1} \quad (1)$$

In order to adaptively estimate the wavelet and scale coefficients without applying the Haar wavelet transform to the entire traffic trace, we present three propositions. The first one consists in the estimation of the values of the average energies of the wavelet coefficients $E[W_{j,i}^2]$ with the knowledge of $E[U_{j,i}^2]$.

Proposition 1. *Let $E[U_{j,i}^2]$ and $E[U_{j+1,i}^2]$ be the average energy of the scale coefficients on scales j and j + 1 respectively. The average energy $E[W_{j,i}^2]$ of the coefficients can be calculated by the following equation:*

$$E[W_{j,i}^2] = 2E[U_{j+1,i}^2] - E[U_{j,i}^2] \quad j = 0, \ldots, J-2 \quad (2)$$

Proof. The sum of the average energies of the wavelet and scale coefficients can be written as:

$$E[W_{j,i}^2] + E[U_{j,i}^2] = \frac{1}{N/2^l} \sum_{i=0}^{N/2^l - 1} \left[W_{j,i}^2 + U_{j+1,2i}^2\right] \quad (3)$$

Replacing the wavelet and scale coefficients by those of the scale $j+1$, that is, $U_{j,i} = (1/\sqrt{2}) \cdot (U_{j+1,2i} + U_{j+1,2i+1})$ and $W_{j,i} = (1/\sqrt{2}) \cdot (U_{j+1,2i} - U_{j+1,2i+1})$ [8,10], we have:

$$SUM = \left(\frac{U_{j+1,2i} + U_{j+1,2i+1}}{\sqrt{2}}\right)^2 + \left(\frac{U_{j+1,2i} - U_{j+1,2i+1}}{\sqrt{2}}\right)^2 \quad (4)$$

where $SUM = \sum_{i=0}^{N/2^l - 1} W_{j,i}^2 + U_{j+1,2i}^2$.

Simplifying (4), we have:

$$E[W_{j,i}^2] + E[U_{j,i}^2] = \frac{1}{N/2^l} \sum_{i=0}^{N/2^l - 1} U_{j+1,i}^2 \quad (5)$$

The right-hand side of (5) is equivalent to $2E[U_{j+1,i}^2]$. Thus, isolating $E[W_{j,i}^2]$, we obtain (2), as we shall demonstrate. □

In order to adaptively estimate $E[W_{J-1,i}^2]$ for the average energy of the high order scale coefficient $J-1$, we present the following proposition:

Proposition 2. *Let $X(k)$ be a discrete signal representing a traffic flow in the time instant k. The average energy $E[W_{J-1,i}^2]$ of highest order scale can be adaptively calculated by the following equation:*

$$E[W_{J-1,i}^2]_{k+1} = \begin{cases} E[W_{J-1,i}^2]_k, & \text{mod } (k+1, 2) \neq 0 \\ \dfrac{(k-1)}{2(k+1)} E[W_{J-1,i}^2]_k + \dfrac{(X(k) - X(k+1))^2}{(k+1)}, & \text{o/w} \end{cases} \quad (6)$$

Proof. The Haar wavelet coefficients $W_{J-1,i}$ of the highest order scale $J-1$ can be given by the difference of two sample values of the process $X(k)$ [8]. Thus, the average energy at instant k of the Haar wavelet coefficients $W_{J-1,i}$ can be obtained through the average of every two samples from the process $X(k)$ by the following equation:

$$E[W_{J-1,i}^2]_k = \frac{1}{\lfloor k/2 \rfloor} \sum_{i=0}^{\lfloor k/2 \rfloor} \frac{(X(k) - X(k+1))^2}{2} \quad (7)$$

For $k+1$, equation (7) can be rewritten as:

$$E[W_{J-1,i}^2]_{k+1} = \frac{1}{(k+1)} \sum_{i=0}^{(k+1)/2} (X(k) - X(k+1))^2 \quad (8)$$

Replacing $E[W_{J-1,i}^2]_{k-1}$ into (8), we obtain the following equation:

$$E[W_{J-1,i}^2]_{k+1} = \frac{\left\lfloor \frac{(k-1)}{2} \right\rfloor}{(k+1)} E[W_{J-1,i}^2]_{k-1} + \frac{(X(k) - X(k+1))^2}{(k+1)} \quad (9)$$

Equation (9) provides a recursive estimation of the average energy of the wavelet coefficients in function of two samples of the process $X(k)$. We also have $mod((k-1), 2) = 0$ and mod $(k+1, 2) = 0$, $E[W_{J-1,i}^2]_k = E[W_{J-1,i}^2]_{k-1}$. Thus, we obtain (6), as we shall demonstrate. □

Now, we propose an equation for the average energy of the scaling coefficients $U_{j,i}$ since it is required in (2).

Proposition 3. *Let $X(k)$ be a discrete time process and $E[X(k+1)]$ be its average. The average energy of the scale coefficients $E[U_{j,i}^2]$ on the jth scale can be adaptively calculated by the following equation:*

$$E[U_{j,i}^2]_{k+1} = \frac{\left\lfloor \frac{k}{2^{(J-j}}) \right\rfloor}{\left\lfloor \frac{(k+1)}{2^{(J-j)}} \right\rfloor} \mathcal{M}_U + \frac{(X(k+1) + \delta)^2}{\left\lfloor \frac{(k+1)}{2^{(J-j)}} \right\rfloor 2^{(J-j)}} \quad (10)$$

where

$$\mathcal{M}_U = \begin{cases} E[U_{j,i}^2]_{k+1}, & \text{if mod } (k+1, 2^{(J-j)}) = 0 \\ \mathcal{M}_U, & \text{if mod } (k+1, 2^{(J-j)}) \neq 0 \end{cases} \quad (11)$$

and

$$\delta = (2^j - mod(k, 2^j) + 1) E[X(k+1)] \quad (12)$$

Proof. The Haar transform of a process can be obtained by a recursive procedure [8]. The first-level scale coefficients $U_{J-1,i}$ correspond to the sum of the sequence of two samples of $X(k)$ divided by $\sqrt{2}$. The second level scale coefficients $U_{J-2,i}$ is equivalent to the sum of the sequence of 4 samples of $X(k)$ divided by $\sqrt{4}$ and so on. Then, a general expression for $U_{j,i}$ can be written as:

$$U_{j,i} = \frac{1}{\sqrt{2^{(J-j)}}} \sum_{t=i2^j}^{i2^j + 2^{(J-j)} - 1} X(k) \quad (13)$$

Let $\mathbf{U}_j$ be the vector containing all the $U_{j,i}$ of scale j, which denotes the network traffic trace aggregated in scale $m = 2^{J-j}$. Then, it can be stated that $E[U_{j,i}^2]_k$ is the average of the second moment of the traffic trace $X(k)$ aggregated in the scale m, at the instant of time $k = n \cdot m$, that is:

$$E[U_{j,i}^2]_k = \frac{1}{\lfloor k/m \rfloor} \sum_{t=0}^{\lfloor k/m \rfloor} \frac{(X_t^m)^2}{2^{(J-j)}} \quad (14)$$

where X_t^m represents the process $X(k)$ aggregated in the scale m.

The value of $E[U_{j,i}^2]_k$ for a time instant before it is completed a new window can be given by the average energy value in the previous window plus δ. The variable δ represents the number of remaining samples of $X(k)$ to a complete time window at instant k. Thus, we have:

$$E[U_{j,i}^2]_{k+1} = \frac{\sum_{t=0}^{\left\lfloor \frac{k}{2^{(J-j)}} \right\rfloor} \left[\frac{(X_t^m)^2}{2^{(J-j)}} + \frac{(X(k+1)+\delta)^2}{2^{(J-j)}} \right]}{\left\lfloor \frac{(k+1)}{2^{(J-j)}} \right\rfloor} \quad (15)$$