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Parameter estimation of coupled polynomial phase and sinusoidal FM signals

Igor Djurović[a,b,*], Pu Wang[c], Marko Simeunović[b,d], Philip V. Orlik[c]

[a] University of Montenegro, Electrical Engineering Department, Cetinjski put bb, Podgorica 81 000, Montenegro
[b] Institute for Cutting Edge Information and Communication Technologies, Dž ordža Vašingtona 66/354, Podgorica 81000, Montenegro
[c] Mitsubishi Electric Research Laboratories (MERL), 201 Broadway, Cambridge, MA 02139, USA
[d] University of Donja Gorica, Faculty for Information Systems and Technologies, Oktoih 1, Podgorica 81 000, Montenegro

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ABSTRACT

This paper considers parameter estimation of a new coupled mixture of polynomial phase signal (PPS) and sinusoidal frequency modulated (FM) signal, recently introduced for industrial systems such as linear electromagnatic encoders. Compared with both conventional PPS-only and independent mixture models, the coupled mixture one captures the coupling between the sinusoidal FM frequency and the PPS parameters induced by structural system configurations. In this paper, we are particularly interested in estimating phase parameters of the coupled mixture signal at low signal-to-noise ratios (SNRs). Specifically, we propose a three-stage approach consisting of instantaneous frequency (IF) extraction (e.g., the short-time Fourier transform) and refining steps that reduce the bias introduced by the IF estimation and the mean-squared errors (MSEs) up to the Cramér-Rao bound (CRB). The proposed method is numerically compared with an existing phase-based approach as well as corresponding CRBs in terms of the empirical MSE. The results show that, compared with the phase-based approach, the proposed method can significantly lower the SNR threshold. The convergence of the measured MSEs from the initial stage to the latter refining stages is also numerically evaluated.

1. Introduction

Parameter estimation of *pure* polynomial phase signals (PPSs) from a finite number of samples is a fundamental problem in many applications, including radar, sonar, communications, acoustics and optics [1–17]. A generalized signal model is an *independent* mixture of PPS and sinusoidal frequency modulated (FM) signal referred to as the hybrid sinusoidal FM-PPS [18–23]. One motivation for studying this kind of signal comes from Doppler radar systems. When a target is moving in a dynamic motion, the resulting signal can be modeled as the pure PPS with parameters associated to the kinematic target parameters. For instance, the initial velocity and acceleration are proportional to the first- and second-order phase parameters, respectively. On the other hand, rotating parts (e.g., rotating blades of a helicopter) and target vibration introduce the sinusoidal FM component [18–20]. With both effects, the matched filter outputs follow the independent mixture signal model.

Motivated by real-world applications, e.g., contactless electromagnetic (EM) encoders, a new *coupled* mixture model of the PPS and sinusoidal FM signal is proposed in [24]. Specifically, the coupling is introduced to express the sinusoidal FM frequency as a function of the PPS parameters. The Cramér-Rao bounds (CRB) for parameter estimation has been established in the same paper. Compared with the independent mixture model, the coupled one can lead to lower bounds for estimating the motion-related PPS parameters as the coupled sinusoidal FM frequency provides additional inference for the PPS parameters. As a first attempt, Wang et al. [25] proposed an instantaneous phase-based method using a phase unwrapping technique followed by a nonlinear coupled least square method, referred to as the PULS. It was shown that the PULS method is unbiased and its estimation performance can approach to the CRB at relatively high signal-to-noise ratio (SNR). However, the PULS method exhibits a high SNR threshold[1] especially for a small number of samples.

In this paper, a parameter estimation method is proposed for the coupled mixture signal at low SNRs. Specifically, we propose

* Corresponding author.
E-mail addresses: igordj@ac.me (I. Djurović), pwang@merl.com (P. Wang), marko.simeunovic@udg.edu.me (M. Simeunović), porlik@merl.com (P.V. Orlik).

[1] The SNR threshold is defined as an SNR value below which the mean-squared error (MSE) of the parameter estimate rapidly deviates from the CRB.

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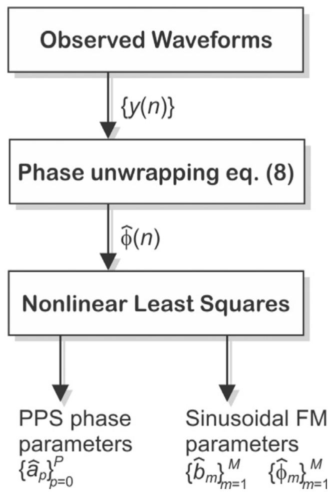

Fig. 1. The flowchart of the phase-based PULS method in [25].

a three-stage approach which features an instantaneous frequency (IF) extraction by using the short-time Fourier transform (STFT) and refining stages to reduce the estimation bias introduced by the initial step and to further push the MSEs towards the CRBs. Moreover, the proposed method is extended to the coupled mixture signal with aliasing spectrum. Further, it is numerically compared with the PULS method via extensive Monte-Carlo simulations. We also show the convergence of the MSEs towards corresponding CRBs when the proposed method moves from the initial stage to the latter refining stages.

The remainder of this paper is organized as follows. Section 2 reviews a specific application which motivates the study of the coupled mixture model, defines the mathematical model, and formulates the problem of interest. Section 3 briefly overviews existing parameter estimation methods and established CRBs. The proposed estimator is introduced in details in Section 4. Numerical examples and performance comparisons are provided in Section 5, followed by a summary in Section 6.

2. A coupled mixture signal model

This section reviews a specific application that motivates the study of the coupled mixture model and compares it with two existing PPS models.

2.1. Linear EM encoders

Accurate speed sensing is highly desired in contactless encoder systems used for motion/position monitoring. Among others, optical, electric, magnetic and EM encoders are commonly used in applications such as auto-tuning drives, smart conveyors, and kit motors [26–30]. Compared with other types of encoders, EM encoders may provide robust sensing capability of position and motion in harsh operating environments, e.g., moisture, heat, vibration and smoke.

Referring to Fig. 1 of [24], the EM encoder normally consists of a stationary scale and a moving readhead, or vice versa. The source transceivers are mounted on the moving readhead with a distance of r to the scale platform. Uniformly spaced reflectors, e.g., rectangular bars, are installed on the scale platform to constitute a spatial period with an inter-reflector spacing of h. The position encoding is achieved by observing the same reflected EM signals at two spatial positions which are separated with a distance of h. Finer position encoding is enabled by detecting the phase changes of two spatial positions (with a distance change less than h) with respect to a full radian period of 2π (corresponding to a distance change of h). Generally, the baseband signals reflected from the spatially periodic linear scale can be written as

$$x(d) = Ae^{j2\pi\left[\frac{d}{h}+\sum_{m=1}^{M} b_m \sin\left(\frac{2\pi m d}{h}+\phi_m\right)+\psi_0\right]}, \tag{1}$$

where A is the unknown amplitude, d is the axial position index of the moving readhead, $b_m > 0$ and ϕ_m are the modulation index and, respectively, the initial phase of the m-th sinusoidal FM component, M is the number of sinusoidal FM components in the phase, and ψ_0 is the initial phase. The first phase term is due to the phase change proportional to the inter-reflector spacing of h. Meanwhile, the second term is, induced by the spatially periodic reflectors, the motion-related sinusoidal FM component. From (1), we have $x(d) = x(d + lh)$, where l is an integer. That is the moving readhead *sees* exactly the same reflected waveforms at two axial positions which are at a distance of h apart from each other.

With a sampling interval of ΔT and assuming that the readhead moves at an initial velocity of v_0 and an acceleration of a, we can transform the position index to the discrete-time index via $d = v_0 t + at^2/2|_{t=n\Delta T} = v_0 n\Delta T + a(n\Delta T)^2/2, n = n_0, \cdots, n_0 + N - 1$ with n_0 and N denoting the initial sampling index and the number of total samples, respectively. As a result, the discrete-time reflected signal for the constantly accelerating readhead is given as

$$x(n) = Ae^{j2\pi\left[\frac{v_0 n\Delta T + a(n\Delta T)^2/2}{h}+\psi_0\right]} \times e^{j\sum_{m=1}^{M} 2\pi\left[b_m \sin\left(2\pi m\frac{v_0 n\Delta T + a(n\Delta T)^2/2}{h}+\phi_m\right)\right]}. \tag{2}$$

Note that the sinusoidal FM frequency is now a function of the motion-related phase parameter (e.g., v_0 and a) of the moving readhead.

2.2. The coupled mixture of PPS and sinusoidal FM signal

For more dynamic motions of the readhead, higher-order phase terms may appear in the reflected signal. For instance, if the acceleration is time-varying, a third-order phase term (on t^3) may be required to model the reflected signal, i.e., $d = v_0 t + at^2/2 + gt^3/6$ where g denotes the acceleration rate. To generalize the model, a *coupled* mixture of the PPS and sinusoidal FM signal is defined as:

$$x(n) = Ae^{j2\pi\left[\sum_{p=0}^{P}\frac{a_p n^p}{p!}+\sum_{m=1}^{M} b_m \sin(2\pi m f_0(n;\mathbf{a})n+\phi_m)\right]}, \tag{3}$$

where the fundamental sinusoidal FM frequency f_0 is now coupled with the PPS phase parameters $\mathbf{a}\triangle=[a_1, \cdots, a_P]^T$, except the initial phase term a_0. Depending on applications, the coupling function $f_0(n; \mathbf{a})$ can be either nonlinear or linear with respect to $\{a_p\}_{p=1}^{P}$. In the case of linear encoders, we have $f_0(n; \mathbf{a}) = c_0 \sum_{p=1}^{P} a_p n^{p-1}/p!$ with c_0 denoting a *known* scaling factor.

To see how the above example of linear EM encoders fits into the coupled mixture of (3), we can establish the following variable changes between (2) and (3)

$$b_m = b_m, \quad a_0 = \psi_0, \quad a_1 = \frac{v_0\Delta T}{h}, \quad a_2 = \frac{a(\Delta T)^2}{h},$$
$$f_0(n;\mathbf{a}) = \frac{v_0\Delta T}{h} + \frac{a(\Delta T)^2}{h}n/2 = c_0(a_1 + a_2 n/2), \tag{4}$$

with $c_0 = 1$, $\mathbf{a} = [a_1, a_2]^T$, and the PPS order of $P = 2$.

2.3. Comparison of signal models

The coupled mixture model given by (3) is distinct from the independent mixture model [18–22]

$$x(n) = Ae^{j2\pi\left[\sum_{p=0}^{P}\frac{a_p n^p}{p!}+\sum_{m=1}^{M} b_m \sin(2\pi m f_0 n+\phi_m)\right]},$$