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Gravitational Clustering: A simple, robust and adaptive approach for distributed networks

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ABSTRACT

Distributed signal processing for wireless sensor networks enables that different devices cooperate to solve different signal processing tasks. A crucial first step is to answer the question: who observes what? Recently, several distributed algorithms have been proposed, which frame the signal/object labelling problem in terms of cluster analysis after extracting source-specific features, however, the number of clusters is assumed to be known. We propose a new method called gravitational clustering to adaptively estimate the time-varying number of clusters based on a set of feature vectors. The key idea is to exploit the physical principle of gravitational force between mass units: streaming-in feature vectors are considered as mass units of fixed position in the feature space, around which mobile mass units are injected at each time instant. The cluster enumeration takes advantage of the fact that the highest attraction on the mobile mass units is exerted by regions with a high density of feature vectors, i.e., gravitational clusters. By sharing estimates among neighboring nodes via a diffusion-adaptation scheme, cooperative and distributed cluster enumeration is achieved. Numerical experiments concerning robustness against outliers, convergence and computational complexity are conducted. The application to distributed cooperative multi-view camera networks illustrates the applicability to real-world problems.

1. Introduction

A recent and emerging research direction in distributed signal processing for wireless sensor networks (WSNs) is that of enabling cooperation among multiple heterogeneous devices dedicated to solve different signal processing tasks [1–5]. A crucial first step towards this so-called multiple devices multiple tasks (MDMT) paradigm is to answer the question: who observes what? [6–9]. For example, to arrive at a node-specific speech signal enhancement [10–12], all relevant speech sources must be uniquely labelled throughout the wireless acoustic sensor network [7]. Similarly, distributed node-specific image/video enhancement requires the common labelling of all objects within a camera network [6].

To illustrate the challenging requirements for such labelling methods, consider for example a video enhancement setup, where multiple users film a nonstationary scene from different angles using their camera equipped portable devices. Each user has its own dedicated signal processing task, e.g. enhancing a specific object of interest. No prior information, such as positions of devices, registration of views or number of objects in the scene, is available, streaming-in data must be processed sequentially and little is known about the distribution of the data. Further, a central computing unit (fusion center) is not available and communication (range, bandwidth) and computation capabilities (memory, computing power), as well as battery power may be limited.

Recently, several distributed algorithms have been proposed, which frame the labelling problem in terms of cluster analysis after extracting source-specific features [6–8,13,14]. Various methods have been proposed for distributed data clustering, e.g., [14–27]. However, a significant drawback of common clustering algorithms is that the number of clusters has to be known a priori. In real scenarios, this information is not always available [28] or the number of clusters might be chosen improperly. Also, in a sensor network the number of clusters may change over time in a non-stationary scenario.

To the best of our knowledge, distributed cluster enumeration has only been addressed in [29], which serves here as a benchmark algorithm. For the single-node case, the question of inferring the number of clusters from the observations has been intensively studied [30–55]. However, most of these approaches are of high computational complexity, need to make prior assumptions on the data distribution or do not allow for an adaptive processing without the need to re-run the entire algorithm. Therefore, these meth-

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ods are not suitable for the above-described object labelling task in MDMT networks.

The aim of this research is to adaptively estimate the time-varying number of clusters based on a set of streaming-in feature vectors. The proposed method is designed to be

1. *Adaptive* - to a changing number of objects/sources.
2. *Robust* - against outliers in the feature vectors, or in general against unknown non-spherical and possibly heavy tailed distributions of the estimated features.
3. *Distributed* - so as to operate in a decentralized WSN, e.g., based on the diffusion-principle [14,56].
4. *Sequential* - so that the estimate of the number of clusters is continuously updated for streaming-in data without the need to re-run the entire algorithm.
5. *Computationally simple* - in order to be applicable in a real WSN.

1.1. Original contributions

A robust gravitational clustering (GC) algorithm is proposed which works for single-node and cooperative in-network clustering. The key idea is to exploit the physical principle of gravitational force between mass units. Streaming-in feature vectors are considered as mass units of a fixed position in the feature space, around which mobile mass units are injected at each time instant. The cluster enumeration takes advantage of the fact that the highest attraction on the mobile mass units is exerted by regions with a high density of feature vectors, i.e., gravitational clusters. The masses of mobile units are combined when they are in a close vicinity of each other and a threshold on the combined mass(es) serves as a detector for a cluster. Since the algorithm updates the estimate of the number of clusters based on streaming-in feature vectors, it is adaptive and suitable for non-stationary scenarios with time-varying numbers of clusters. By sharing estimates among neighboring nodes via a diffusion-adaptation scheme, cooperative and distributed cluster enumeration for sensor networks is achieved. An algorithm to determine a suitable radius for the combination vicinity is also proposed. An extensive simulation-based performance analysis is provided that investigates the clustering performance for single-node and multi-node cluster enumeration. Herein, aspects such as robustness against outliers, computational cost and convergence are studied. The applicability of the GC algorithm is illustrated for a use case of labelling moving objects in a synthetic 3-D multi camera network.

1.2. Related work

The idea to cluster data based on the law of gravity was first proposed by Wright in [57] where clustering is performed by moving and merging the data points based on gravitational force until one final cluster remains. This approach has been extended in some works, e.g., [58,59], which consider multiple clusters. An overview of existing methods which exploit the gravitational principle is provided in [59]. In these methods, a decay term prevents that all samples conflate into one big cluster, or a threshold is set that determines up to which distances clusters should stay separated and which clusters may merge. This requires prior knowledge about the data, e.g., the minimum distance between the clusters or the distance of the data points from their corresponding cluster centroid, in order to assure adequate performance. This kind of information is not always available in practice. Another drawback is that one cannot draw inferences from the resulting clusters about the actual positions of the cluster centroids since the data points (and therefore the cluster centroids) change their positions because of their mutual attraction. Further, such a procedure makes it difficult to adapt to changes in the scenario without the need to re-run the algorithm.

1.3. Notation

The following notation is used: vectors are denoted by bold small letters $\mathbf{a}$ and matrices by bold capital letters $\mathbf{A}$. All vectors are defined as column vectors. Sets are denoted by calligraphic letters $\mathcal{A}$ with $|\cdot|$ indicating the cardinality of a set, the notation $\mathcal{A}\setminus i$ describes the resulting set after excluding element i from $\mathcal{A}$ while $\|\cdot\|$ denotes the Euclidean norm of a vector. The superscript $^\top$ denotes the transpose operator, diag$\{\mathbf{x}\}$ is a diagonal matrix with $\mathbf{x} = (x_1, \ldots, x_M)^\top$ as diagonal elements, and $\mathbf{I}_M$ stands for an $M \times M$ identity matrix.

1.4. Organization

Section 2 provides the problem formulation and data model. Section 3 is dedicated to the proposal of our GC algorithm, while Section 4 provides an extensive Monte-Carlo simulation study. Section 5 applies the proposed method to a multi-view camera network and Section 6 concludes the paper and provides future research directions.

2. Problem formulation, signal model and aims

We consider a network of J nodes whose topology is described by a graph with nodes indexed by $j \in 1, \ldots, J$. The neighborhood of node j, denoted as $\mathcal{B}_j$, is the set of nodes, including j, that node j exchanges information with, and $|\mathcal{B}_j|$ denotes its associated cardinality. Each observation is assumed to belong to a certain cluster $\mathcal{C}_k$ with $k \in 1, \ldots, K$ denoting the label of the given cluster. The total number of clusters K is assumed to be unknown and might change over time. Each cluster is described by a set of application-dependent descriptive statistics (features).

The feature estimation process is an application-specific research area of its own (see, e.g., [6,7]) and is not the focus of this article, where we seek for a generic adaptive and robust cluster enumeration method. It is assumed that the features have already been extracted and the uncertainty within each cluster k can be modeled by a probability distribution, e.g., the Gaussian. Further, we account for gross estimation errors in the feature extraction process that we consider as outliers, thus arriving at the following observation model for feature vectors of node j at time instant $t = 1, \ldots, N$:

$$\mathbf{d}_{kj}(t) = \boldsymbol{w}_k(t) + \boldsymbol{n}_{kj}(t). \tag{1}$$

Here, $\boldsymbol{w}_k(t)$ denotes the class centroid, $\boldsymbol{n}_{kj}(t)$ represents a stochastic, cluster-specific uncertainty term of unspecified distribution with associated covariance matrix $\boldsymbol{\Sigma}_{jk} \in \mathbb{R}^{q\times q}$, and $\boldsymbol{d}_{kj}(t), \boldsymbol{w}_k(t), \boldsymbol{n}_{kj}(t) \in \mathbb{R}^{q\times 1}$. For reasons of visual clarity, we drop the index k in the feature vectors and refer to them as $\boldsymbol{d}_j(t)$.

The aim of this research is to estimate the time-varying number of clusters $K(t)$ and class centroids $\boldsymbol{w}_k(t)$ based on a set of streaming-in feature vectors $\mathbf{d}_j(t)$. The proposed method should be *adaptive, robust, distributed, sequential and computationally simple*, as defined in Section 1.

3. Description of the proposed Gravitational Clustering (GC) Algorithm

GC is based on Isaac Newton's law of universal gravitation which relates the force $\boldsymbol{f}$ between two mass units with masses m_1 and m_2 and distance $\|\boldsymbol{r}_{12}\|$ by

$$\boldsymbol{f}_{12} = -\boldsymbol{f}_{21} = g \cdot m_1 \cdot m_2 \cdot \frac{\boldsymbol{e}_{12}}{\|\boldsymbol{r}_{12}\|^2},$$

where $g = 6.67408 \times 10^{11}$ m^3 kg^{-1} s^{-2} is the gravitational constant and $\boldsymbol{e}_{12}$ denotes the unit vector which points from mass unit 1