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Brief paper

Plug-and-play model predictive control based on robust control invariant sets[☆]

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ABSTRACT

We consider the problem of designing decentralized controllers for large-scale linear constrained systems composed by a number of interacting subsystems. As in Riverso et al. (2013b), (i) the design of local controllers requires limited transmission of information from other subsystems and (ii) the addition/removal of a subsystem triggers the design of local controllers for child subsystems only. These properties enable Plug-and-Play (PnP) operations, and we show how to perform them while preserving global stability of the origin and constraint satisfaction. We improve several aspects of the PnP design procedure proposed in Riverso et al. (2013b) and, using recent results in the computation of Robust Control Invariant (RCI) sets, we show that all critical steps in the design of a local controller can be solved through Linear Programming (LP). Finally, an application of the proposed design procedure to a large-scale mechanical system is presented.

1. Introduction

The ever-increasing complexity and size of process plants, manufacturing systems, transportation systems and power networks has triggered a renewed interest in decentralized and distributed control schemes, that have been studied since the 1970s for unconstrained models (Lunze, 1992; Šiljak, 1991). In a nutshell, decentralized control assumes the overall plant is represented through the coupling of several subsystems for which local regulators are designed. The main advantages of this architecture are that the computation of control variables for different subsystems is parallelized and only communication between a subsystem and its local controller is required. Similar remarks also apply to distributed controllers where local controllers can also exchange information through a communication network.

In the last years, many decentralized/distributed MPC (De/DiMPC) schemes have been proposed (Scattolini, 2009), in view of the possibility of coping with constraints on system variables besides guaranteeing stability, robustness, and global optimality (Rawlings & Mayne, 2009). Available DiMPC methods span from cooperative (Stewart, Venkat, Rawlings, Wright, & Pannocchia, 2010) to non-cooperative, which require limited computational load, memory, and transmission of information (Camponogara, Jia, Krogh, & Talukdar, 2002; Farina & Scattolini, 2012; Riverso & Ferrari-Trecate, 2012; Trodden & Richards, 2010). One of the main problems of existing De/DiMPC approaches is the need of a centralized off-line design phase. In the context of large-scale systems, this can be a severe limitation because a global model of the system can be very hard or costly to obtain. Moreover, in several examples of systems of systems, units frequently enter and leave a network (Samad & Parisini, 2011) making it impractical to retune the overall controller in a centralized fashion. In these cases, a decentralized design based on local computational resources is the only viable approach.

In Riverso, Farina, and Ferrari-Trecate (2013b) we proposed a novel controller synthesis procedure based on the PnP paradigm (Stoustrup, 2009). PnP design, besides synthesis decentralization, requires limited information transmission for the synthesis of local

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controllers when subsystems are added or removed. Furthermore, the complexity of controller design and implementation, for a given subsystem, scales with the number of its parent subsystems only.

As in Riverso et al. (2013b), we propose a PnP design procedure hinging on tube MPC (Raković & Mayne, 2005) for handling coupling among subsystems, and aim at stabilizing the origin of the whole closed-loop system while guaranteeing satisfaction of constraints on local inputs and states. However, we advance the design procedure in Riverso et al. (2013b) in two main directions: (I) while in Riverso et al. (2013b) the design of local controllers requires the solution to nonlinear optimization problems, in this paper, using regulators based on RCI sets (Raković & Baric, 2010; Raković & Mayne, 2005), only the solution to Linear Programming (LP) problems is needed; (II) in Riverso et al. (2013b) stability requirements are fulfilled imposing an aggregate sufficient small-gain condition for networks, while in this paper we resort to set-based conditions that are usually less conservative. As for any decentralized synthesis procedure for general linear systems without a special structure, our method involves some degree of conservativity (Bakule & Lunze, 1988). More specifically, it requires that coupling between subsystems giving rise to loops is small enough. The potential application of our method to real-world systems is assessed through examples. In Riverso, Farina, and Ferrari-Trecate (2012, 2013a) we present an application of PnP-DeMPC to frequency control in power networks and compare results with those achievable by centralized MPC and the control scheme in Riverso et al. (2013b). In particular, our new controller outperforms the PnP controllers described in Riverso et al. (2013b). In this paper we highlight computational advantages brought about by our method by considering the control of a large array of masses connected by springs and dampers.

The paper is structured as follows. The design of decentralized controllers is introduced in Section 2. In Section 3 we discuss how to design local controllers by solving LP problems and in Section 4 we describe PnP operations. Sections 5 and 6 are devoted to a numerical example and some conclusions, respectively. Generalizations of PnP-DeMPC to distributed control architectures are given in Riverso et al. (2012). A preliminary version of this work has been presented at the 52nd IEEE Conference on Decision and Control (Riverso et al., 2013a).

Notation. We use $a : b$ for the set of integers $\{a, a+1, \dots, b\}$. The column vector with s components $v_1, \dots, v_s$ is $\mathbf{v} = (v_1, \dots, v_s)$. The function $\mathrm{diag}(G_1, \dots, G_s)$ denotes the block-diagonal matrix composed by s block G_i, $i \in 1 : s$. The symbols $\oplus$ and $\ominus$ denote the Minkowski sum and difference, respectively, i.e. $A = B \oplus C$ if $A = \{a : a = b + c$, for all $b \in B$ and $c \in C\}$ and $A = B \ominus C$ if $a \oplus C \subseteq B$, $\forall a \in A$. Moreover, $\bigoplus_{i=1}^{s} G_i = G_1 \oplus \cdots \oplus G_s$. For $\rho > 0$, $B_\rho(z) = \{x \in \mathbb{R}^n : \|x - z\| \le \rho\}$ where $\|\cdot\|$ is the Euclidean norm in $\mathbb{R}^n$. Given a set $\mathbb{X} \subset \mathbb{R}^n$, $\mathrm{convh}(\mathbb{X})$ denotes its convex hull. The symbol $\mathbf{1}$ denotes a column vector of suitable dimension with all elements equal to 1.

Definition 1 (*RCI Set*)**.** Consider the discrete-time Linear Time-Invariant (LTI) system $x(t+1) = Ax(t) + Bu(t) + w(t)$, with $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $w(t) \in \mathbb{R}^n$ and subject to constraints $u(t) \in \mathbb{U} \subseteq \mathbb{R}^m$ and $w(t) \in \mathbb{W} \subset \mathbb{R}^n$. The set $\mathbb{X} \subseteq \mathbb{R}^n$ is an RCI set with respect to $w(t) \in \mathbb{W}$, if $\forall x(t) \in \mathbb{X}$ there exists $u(t) \in \mathbb{U}$ such that $x(t+1) \in \mathbb{X}$, $\forall w(t) \in \mathbb{W}$.

2. Decentralized MPC for linear systems

We consider the discrete-time LTI system

$$\mathbf{x}^+ = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ and $\mathbf{u} \in \mathbb{R}^m$ are the state and the input, respectively, at time t and $\mathbf{x}^+$ stands for $\mathbf{x}$ at time $t+1$. The notation $\mathbf{x}(t)$, $\mathbf{u}(t)$ is used only if necessary. The state $\mathbf{x} = (x_{[1]}, \dots, x_{[M]})$ is partitioned into the M vectors $x_{[i]} \in \mathbb{R}^{n_i}$, where $i \in \mathcal{M} = 1 : M$ and $n = \sum_{i \in \mathcal{M}} n_i$. Similarly, $\mathbf{u} = (u_{[1]}, \dots, u_{[M]})$ where $u_{[i]} \in \mathbb{R}^{m_i}$, $i \in \mathcal{M}$ and $m = \sum_{i \in \mathcal{M}} m_i$. Let the ith subsystem be given by

$$\Sigma_{[i]} : x^+_{[i]} = A_{ii} x_{[i]} + B_i u_{[i]} + w_{[i]} \tag{2}$$

$$w_{[i]} = \sum_{j \in \mathcal{N}_i} A_{ij} x_{[j]} \tag{3}$$

where $A_{ij} \in \mathbb{R}^{n_i \times n_j}$, $i, j \in \mathcal{M}$, $B_i \in \mathbb{R}^{n_i \times m_i}$ and $\mathcal{N}_i = \{j \in \mathcal{M} : A_{ij} \neq 0, i \neq j\}$ is the set of parents to subsystem i. Subsystems Σ_i are state coupled and input decoupled. Moreover, under the following assumption, they are equivalent to (1).

Assumption 1. Matrix $\mathbf{A}$ is composed by blocks A_{ij}, $i, j \in \mathcal{M}$ and $\mathbf{B} = \mathrm{diag}(B_1, \dots, B_M)$.

We equip subsystems $\Sigma_{[i]}$, $i \in \mathcal{M}$ with the constraints

$$x_{[i]} \in \mathbb{X}_i, \qquad u_{[i]} \in \mathbb{U}_i. \tag{4}$$

Moreover, we define the sets $\mathbb{X} = \prod_{i \in \mathcal{M}} \mathbb{X}_i$, $\mathbb{U} = \prod_{i \in \mathcal{M}} \mathbb{U}_i$ and add to system (1) the constraints

$$\mathbf{x} \in \mathbb{X}, \qquad \mathbf{u} \in \mathbb{U}. \tag{5}$$

We consider the following assumptions.

Assumption 2. The matrix pairs (A_{ii}, B_i) $\forall i \in \mathcal{M}$ are controllable.

Assumption 3. Constraints $\mathbb{X}_i$ and $\mathbb{U}_i$, $i \in \mathcal{M}$ are compact and convex polytopes containing the origin in their nonempty interior.

For the design of suitable decentralized regulators, local controllers are designed following the tube MPC scheme in Raković and Mayne (2005) (see also Rawlings & Mayne, 2009). To this purpose, we treat $w_{[i]} \in \mathbb{W}_i = \bigoplus_{j \in \mathcal{N}_i} A_{ij} \mathbb{X}_j$ as a disturbance and define the nominal (unperturbed) system $\hat{\Sigma}_{[i]}$ as

$$\hat{\Sigma}_{[i]} : \hat{x}^+_{[i]} = A_{ii} \hat{x}_{[i]} + B_i v_{[i]} \tag{6}$$

where $v_{[i]} \in \mathbb{R}^{m_i}$ is the input. We want to confine $x_{[i]}$ in a tube of section $\mathbb{Z}_i$ centered in $\hat{x}_{[i]}$, i.e. to obtain that

$$x_{[i]}(0) \in \hat{x}_{[i]}(0) \oplus \mathbb{Z}_i \Rightarrow x_{[i]}(t) \in \hat{x}_{[i]}(t) \oplus \mathbb{Z}_i, \quad \forall t \ge 0. \tag{7}$$

This can be achieved (Raković & Mayne, 2005) if (a) $\mathbb{Z}_i$ is a nonempty RCI set for the constrained subsystem (2) with respect to the disturbance w_i; (b) for $\bar{x} = \hat{x}$ the local controller

$$\mathcal{C}_{[i]} : u_{[i]} = v_{[i]} + \bar{\kappa}_i (x_{[i]} - \bar{x}_{[i]}) \tag{8}$$

is used, where $\bar{\kappa}_i : \mathbb{Z}_i \to \mathbb{U}_i$ is any feedback control law[2] guaranteeing $x_{[i]} \in \mathbb{Z}_i \Rightarrow x^+_{[i]} \in \mathbb{Z}_i$, $\forall w \in \mathbb{W}$.

Following Raković and Mayne (2005), in (8) we set

$$v_{[i]}(t) = v_{[i]}(0|t), \qquad \bar{x}_{[i]}(t) = \hat{x}_{[i]}(0|t) \tag{9}$$

where $v_{[i]}(0|t)$ and $\hat{x}_{[i]}(0|t)$ are optimal values of the variables $v_{[i]}(0)$ and $\hat{x}_{[i]}(0)$, respectively, appearing in the MPC-i problem $\mathbb{P}_i^N(x_{[i]}(t))$

$$\min_{\substack{v_{[i]}(0:N_i-1) \\ \hat{x}_{[i]}(0)}} \sum_{k=0}^{N_i-1} \ell_i(\hat{x}_{[i]}(k), v_{[i]}(k)) + V_{f_i}(\hat{x}_{[i]}(N_i)) \tag{10a}$$

$$x_{[i]}(t) - \hat{x}_{[i]}(0) \in \mathbb{Z}_i \tag{10b}$$

2 Definition 1 guarantees the existence of a function $\bar{\kappa}_i$.