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An impulse-to-impulse discrete-time mapping for a time-delay impulsive system☆

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ABSTRACT

It is shown that an impulsive system with a time-delay in the continuous part can be equivalently represented by discrete dynamics under less restrictive conditions on the time-delay value than considered previously.

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1. Introduction

This note deals with the reduction to discrete-time dynamics of a (hybrid) system comprised of a time-delay continuous part under impulsive feedback. The delay-free system was considered in Churilov, Medvedev, and Shepeljavyi (2009) as a model of endocrine regulation implementing the principles of impulsive control (see e.g. Gelig and Churilov (1998)). Further, the initial model was augmented with a time delay in the continuous part in Churilov, Medvedev, and Mattsson (2012), Churilov, Medvedev, and Mattsson (2013) and Churilov, Medvedev, and Mattsson (2014). The relevance of this model to biological data is discussed in Mattsson and Medvedev (2013). The delay value was previously assumed to be strictly less than the least time interval between two consecutive firing times of the impulsive feedback, the relationship which does not always hold in endocrine applications.

The main approach to the analysis of the impulsive system in hand is its reduction to a discrete-time model, mapping the continuous part state vector from one impulse of the feedback to another. Then the resulting nonlinear discrete dynamics are treated by conventional methods. Below, a reduction procedure covering longer time delays in the continuous part than considered previously is suggested. Moreover, Theorem 1 of this paper improves on the results of Churilov et al. (2014) by providing a simpler form of the discrete-time model.

The structure of this note is as follows. First, the notions of finite-dimension reducible time-delay systems and the impulsive Goodwin–Smith model are recalled. Then a pointwise mapping propagating the continuous dynamics of the impulsive system through the firing times of the impulsive feedback is derived, constituting the contribution of the paper.

2. FD-reducible linear time delay systems

Consider a continuous linear time-delay system

$$\dot{x} = A_0 x(t) + A_1 x(t-\tau), \tag{1}$$

where $x(t) \in \mathbb{R}^p$, $A_0, A_1 \in \mathbb{R}^{p\times p}$, and τ is a constant time delay for $t \geqslant 0$ with an initial (vector) function $x(t) = \varphi(t)$, $-\tau \leqslant t < 0$.

Definition 1 (*Churilov et al., 2012*)**.** Time-delay system (1) is called *finite-dimension reducible* (FD-reducible), if there exists a constant matrix $D \in \mathbb{R}^{p\times p}$ such that any solution $x(t)$ of (1) defined for $t \geqslant 0$ satisfies the linear differential equation $\dot{x} = Dx$ for all $t \geqslant \tau$.

FD-reducibility means that the solutions of time-delay system (1) are indistinguishable from those of a finite-dimensional system of order p on the time interval $[\tau, +\infty)$. The proposition below summarizes the essential properties of FD-reducible systems.

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Proposition 1 (Churilov et al., 2013). *Either of the statements* (i)–(ii) *equivalently implies FD-reducibility of system* (1)*:*

(i) *The matrix coefficients of* (1) *satisfy*

$A_1A_0^kA_1 = 0$ *for all* $k = 0, 1, \ldots, p-1$.

(ii) *There exists an invertible* $p \times p$ *matrix* S *such that*

$$S^{-1}A_0S = \begin{bmatrix} U & 0 \\ W & V \end{bmatrix}, \qquad S^{-1}A_1S = \begin{bmatrix} 0 & 0 \\ \bar{W} & 0 \end{bmatrix}, \tag{2}$$

where the blocks U, V *are square and the sizes of the blocks* W *and* $\bar{W}$ *are equal.*

Moreover, the matrix D *for an FD-reducible system* (1) *is uniquely given by*

$$D = A_0 + A_1 e^{-A_0\tau}. \tag{3}$$

By virtue of (3), Eq. (1) can be rewritten as

$$\dot{x}(t) = Dx(t) + (D - A_0)\left[e^{A_0\tau}x(t-\tau) - x(t)\right]. \tag{4}$$

3. A time-delay impulsive system

Consider an extension of the impulsive Goodwin–Smith model treated in Churilov et al. (2009) to the class of systems with delayed continuous part:

$$\begin{aligned} &\dot{x} = A_0x(t) + A_1x(t-\tau), && y = Cx, \\ &t_{n+1} = t_n + T_n, && x(t_n^+) = x(t_n^-) + \lambda_n B, \\ &T_n = \Phi(y(t_n)), && \lambda_n = F(y(t_n)). \end{aligned} \tag{5}$$

Without loss of generality, assume $t_0 = 0$. Here $B \neq 0$ is a column and C is a row such that $CB = 0$.

The continuous functions $\Phi(\cdot)$, $F(\cdot)$ are bounded with strictly positive lower bounds and finite upper bounds. The latter condition implies that system (5) has no equilibria, Churilov et al. (2009).

Previously, in Churilov et al. (2012) and Churilov et al. (2013), the case of $\inf_y \Phi(y) > \tau$ was addressed so that $T_k > \tau$ for all $k \geqslant 0$. In this paper, a less restrictive condition of

$$2 \inf_y \Phi(y) > \tau$$

is imposed on the time delay value. The latter results in $\Phi(Cx) + \Phi(Cz) > \tau$ for all vectors $x \in \mathbb{R}^p$, $z \in \mathbb{R}^p$. Hence

$$T_k + T_{k-1} > \tau, \quad \text{for all } k \geqslant 1. \tag{6}$$

Define four sets of vector pairs (x, z), where $x \in \mathbb{R}^p$, $z \in \mathbb{R}^p$:

$$\begin{aligned} \Omega_1 &= \{(x,z) : \Phi(Cx) > \tau, \Phi(Cz) > \tau\}, \\ \Omega_2 &= \{(x,z) : \Phi(Cx) < \tau, \Phi(Cz) > \tau\}, \\ \Omega_3 &= \{(x,z) : \Phi(Cx) > \tau, \Phi(Cz) < \tau\}, \\ \Omega_4 &= \{(x,z) : \Phi(Cx) < \tau, \Phi(Cz) < \tau\}. \end{aligned}$$

Obviously, the space $\mathbb{R}^p \times \mathbb{R}^p$ coincides with a union of the closures of Ω_i, $i = 1, 2, 3, 4$.

Introduce now the following four maps for $x, z \in \mathbb{R}^p$:

$$\begin{aligned} Q_1(x) &= e^{D\Phi(Cx)}x + F(Cx)e^{D(\Phi(Cx)-\tau)}e^{A_0\tau}B; \\ Q_2(x) &= e^{D\Phi(Cx)}x + F(Cx)e^{A_0\Phi(Cx)}B, \\ Q_3(x,z) &= Q_1(x) + e^{D\Phi(Cx)}\left[Q_1(z) - Q_2(z)\right], \\ Q_4(x,z) &= Q_2(x) + e^{D\Phi(Cx)}\left[Q_1(z) - Q_2(z)\right]. \end{aligned}$$

Evidently, for the values of x yielding $\Phi(Cx) = \tau$, one has $Q_1(x) = Q_2(x)$ and so $Q_3(x,z) = Q_4(x,z)$. At the same time, if $\Phi(Cz) = \tau$, then $Q_1(z) = Q_2(z)$, so $Q_1(x) = Q_3(x,z)$ and $Q_2(x) = Q_4(x,z)$.

Introduce a function $Q(x,z)$ as $Q(x,z) = Q_i(x)$ for $(x,z) \in \Omega_i$, $i = 1, 2$, and $Q(x,z) = Q_i(x,z)$ for $(x,z) \in \Omega_i$, $i = 3, 4$. If $\Phi(Cx) = \tau$ or $\Phi(Cz) = \tau$, then define $Q(x,z)$ preserving its continuity. In this manner, the function $Q(x,z)$ is defined and continuous in the entire space $\mathbb{R}^p \times \mathbb{R}^p$.

Introduce the shorthand notation $x_n = x(t_n^-)$.

Theorem 1. *Let* $n \geqslant 2$. *Then any solution of* (5) *satisfies the recursion*

$$x_{n+1} = Q(x_n, x_{n-1}). \tag{7}$$

Remark. More precisely, if $T_0 > \tau$, then (7) is valid for $n \geqslant 1$. Otherwise, (7) is valid for $n \geqslant 2$. If an initial function $\varphi(t)$, $-\tau \leqslant t \leqslant 0$ is given, then $x_0 = \varphi(0)$ and the initial points x_1 (if $T_0 > \tau$) or x_1, x_2 (if $T_0 \leqslant \tau$, $T_0 + T_1 > \tau$) can be obtained by direct integration of (5).

Proof of Theorem 1. In a special coordinate basis described by (2), system (1) can be represented as

$$\dot{u} = Uu, \qquad \dot{v} = Wu + Vv + \bar{W}u(t-\tau), \tag{8}$$

with $x^{\mathrm{T}} = [u^{\mathrm{T}}, \; v^{\mathrm{T}}]$, where $\cdot^{\mathrm{T}}$ denotes transpose. Thus D defined by (3) takes the form

$$D = \begin{bmatrix} U & 0 \\ W + \bar{W}e^{-U\tau} & V \end{bmatrix} \quad \text{and} \quad D - A_0 = \begin{bmatrix} 0 & 0 \\ \bar{W}e^{-U\tau} & 0 \end{bmatrix}.$$

Then (4) can be rewritten as

$$\dot{x} = Dx(t) - (D - A_0)\eta(t), \tag{9}$$

where

$$\eta(t) = \begin{bmatrix} u(t) - e^{U\tau}u(t-\tau) \\ * \end{bmatrix}$$

and $*$ stands for any vector of a suitable size. It is convenient now to, without loss of generality, assume that the first equation in (5) is in the form of (8).

Introduce also the partition $B^{\mathrm{T}} = [B_1^{\mathrm{T}}, \; B_2^{\mathrm{T}}]$, where the dimensions of the vectors B_1, B_2 correspond to those of u, v, respectively. One has

$$\begin{aligned} Q_1(x_n) &= e^{DT_n}x_n + \lambda_n e^{D(T_n-\tau)}e^{A_0\tau}B, \\ Q_2(x_n) &= e^{DT_n}x_n + \lambda_n e^{A_0T_n}B. \end{aligned}$$

Define time intervals

$$L_k = \{t : t_k < t < t_{k+1}\}, \quad k = 0, 1, \ldots.$$

Then for $t \in L_k$ one can write

$$u(t) = e^{U(t-t_k)}u(t_k^+) = e^{U(t-t_{k+1})}u(t_{k+1}^-).$$

Consider a solution $x(t)$ on any interval L_n, where $n \geqslant 2$.

Case (i): $(x_n, x_{n-1}) \in \Omega_1$.

Then $T_n > \tau$, $T_{n-1} > \tau$ and

$$t_{n-1} + \tau < t_n < t_n + \tau < t_{n+1}.$$

The argument generally follows (Churilov et al., 2013). As stated above, if $t \in L_n$, then the function $u(t-\tau)$ has only one jump, i.e. at the point $t = t_n + \tau$. Consider $u(t)$, $v(t)$ in the subintervals of L_n, where $u(t-\tau)$ is continuous.

(i-a) Let $t_n < t < t_n + \tau$. Then $t - \tau \in L_{n-1}$ and

$$\begin{aligned} u(t-\tau) &= e^{U(t-\tau-t_n)}u(t_n^-) \\ &= e^{U(t-\tau-t_n)}(u(t_n^+) - \lambda_n B_1) \\ &= e^{-U\tau}u(t) - \lambda_n e^{U(t-\tau-t_n)}B_1. \end{aligned} \tag{10}$$

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