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On the controllability and observability of networked dynamic systems[☆]

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ABSTRACT

Some necessary and sufficient conditions are obtained for the controllability and observability of a networked system with linear time invariant (LTI) dynamics. The topology of this system is fixed but arbitrary, and every subsystem is permitted to have different dynamics. These conditions essentially depend only on transmission zeros of every subsystem and the subsystem connection matrix, which makes them attractive in the analysis and synthesis of a large-scale networked system. As an application, these conditions are utilized to characterize systems whose steady estimation accuracy with the distributed predictor of Zhou (2013) is equal to that of the lumped Kalman filter. It has been made clear that to guarantee this equivalence, the steady update gain matrix of the Kalman filter must be block diagonal.

1. Introduction

In system analysis and synthesis, a fundamental issue is controllability and observability verification. While the former is concerned with possibilities of maneuvering system internal variables through external inputs, the latter is concerned with potentials of estimating them from external measurements. It is now widely known that various important system properties, such as existence of an optimal control, possibilities of properly locating plant poles, convergence of a state estimator, etc., are closely related to this verification (Ferrari, Parisini, & Polycarpou, 2012; Kailath, Sayed, & Hassibi, 2000; van Schuppen et al., 2011; Zhou, Doyle, & Glover, 1996). While this issue has been extensively studied over more than half a century and various criteria have been well developed, difficulties arise when they are straightforwardly applied to a networked system consisting of a huge number of subsystems. To be more specific, it has been recognized that for large-scale networked systems, these criteria are usually computationally prohibitive, and various efforts have recently been put in developing a more computationally efficient one (Hendrickx, Olshevsky, & Tsitsiklis, 2011; Liu, Slotine, & Barabasi, 2011; Notarstefano & Parlangeli, 2013; Siljak, 1978).

Owing to extensive pursuits of numerous researchers, significant developments have been achieved. For example, in Notarstefano and Parlangeli (2013), Laplacian of a grid graph is adopted in verifying controllability/observability of a family of linear dynamic systems. Through some smart graph decompositions, computable necessary and sufficient conditions are derived which can distinguish all the necessary nodes that lead to a controllable/observable dynamic system. In Liu et al. (2011), based on structural controllability and the cavity method, some computationally efficient analytic tools are developed to identify a driver node set. Some interesting results have been observed there, such as driver nodes intend to avoid network hubs, biological regulatory networks are significantly more difficult to be controlled than a social network, etc.

These efforts have greatly advanced analysis and synthesis of large-scale networked systems. Complete settlement of its controllability/observability verification problem, however, still requires further efforts, noting that all the existing methods ask some conditions that may not be easily satisfied in practice. For example, the results of Notarstefano and Parlangeli (2013) are derived under the condition of equal subsystem interaction strength, while Liu et al. (2011) requires precise location knowledge on the zero elements of the plant state transition matrix and controllability is evaluated with a probabilistic metric.

A closely related problem is about controllability/observability of multi-agent systems, which is a special kind of networked

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systems and has attracted extensive attentions over the last decade. Essential issues like robustness of structural controllability, input addition, decentralized controllability, etc., have been investigated respectively in Commault and Dion (2013), Guan, Ji, Zhang, and Wang (2013), Rahimian and Aghdam (2013) and Valcher and Misra (2012). Various interesting results have been obtained and some recent important advances are summarized in Egerstedt, Martini, Cao, Camlibel, and Bicchi (2012). Except Guan et al. (2013) and Valcher and Misra (2012), however, almost all these researches are focused on a system with first order subsystems. Moreover, Valcher and Misra (2012) requires that every subsystem is of single-input and single-output, while Guan et al. (2013) asks identical subsystem dynamics. In addition, rather than computation complexity, these investigations are much more concentrated on subsystem selection, system controllability with communication failures, etc.

In this paper, we re-investigate controllability and observability of a linear time invariant (LTI) plant consisting of several subsystems. Interactions among subsystems are arbitrary except that the subsystem connection matrix (SCM) is time independent, and the required knowledge is restricted to a state space model (SSM) of each subsystem and the SCM. Based on the PBH test, some necessary and sufficient conditions are obtained, which depend essentially only on transmission zeros of its subsystems and its SCM. This characteristic makes these conditions attractive in analyzing and synthesizing large-scale networked systems. Hopefully, these conditions are helpful in input and/or communication structure selection. As an application, situations are discussed under which the distributed state predictor developed in Zhou (2013) has the same steady estimation accuracy as that of the Kalman filter. Some necessary and sufficient conditions are obtained for this equivalence. It has been made clear that in order to guarantee this equivalence, the steady update gain matrix of the Kalman filter must be block diagonal.

The outline of this paper is as follows. At first, Section 2 gives an SSM like representation (SSMR) for networked dynamic systems and some preliminary results. Controllability/observability of a networked dynamic system is investigated in Section 3. Some necessary and sufficient conditions are given in Section 4 for networked systems to have a steady estimation accuracy with the distributed predictor of Zhou (2013) equal to that of the Kalman filter. Section 5 provides two numerical examples illustrating advantages and shortcomings of the obtained theoretical results. Finally, some concluding remarks are given in Section 6. Two Appendices are included. One gives proofs of the technical results, while the other provides subsystem parameters of the first numerical example.

The following notation and symbols are adopted. $\mathbf{diag}\{X_i|_{i=1}^L\}$ denotes a block diagonal matrix with its ith diagonal block being X_i, while $\mathbf{col}\{X_i|_{i=1}^L\}$ the vector/matrix stacked by $X_i|_{i=1}^L$ with its ith row block vector/matrix being X_i. $\{X_{ij}|_{i=1,j=1}^{i=M,j=N}\}$ represents a matrix with $M \times N$ blocks and its ith row jth column block matrix being X_{ij}, while 0_m and $0_{m\times n}$ respectively the m dimensional zero column vector and the $m \times n$ dimensional zero matrix. The superscript T and H are used to denote respectively the transpose and the conjugate transpose of a matrix/vector, and X^TWX or XWX^T is sometimes abbreviated as $(\star)^TWX$ or $XW(\star)^T$, especially when the term X has a complicated expression. $\mathbf{E}\{\star\}$ is used to denote the mathematical expectation of a random variable/matrix. When a time dependent function becomes time independent, its temporal variable is usually omitted.

2. System description and some preliminaries

Consider the following networked system $\mathbf{\Sigma}$ which is constituted from N LTI dynamic subsystems with the dynamics of its ith subsystem $\mathbf{\Sigma}_i$ being described by

$$\begin{bmatrix} x(t+1,i) \\ z(t,i) \\ y(t,i) \end{bmatrix} = \begin{bmatrix} A_{\mathbf{TT}}(i) & A_{\mathbf{TS}}(i) & B_{\mathbf{T}}(i) & 0 \\ A_{\mathbf{ST}}(i) & A_{\mathbf{SS}}(i) & B_{\mathbf{S}}(i) & 0 \\ C_{\mathbf{T}}(i) & C_{\mathbf{S}}(i) & D_{\mathbf{d}}(i) & D_{\mathbf{w}}(i) \end{bmatrix} \begin{bmatrix} x(t,i) \\ v(t,i) \\ d(t,i) \\ w(t,i) \end{bmatrix} \tag{1}$$

while interactions among its subsystems by

$$v(t) = \Phi z(t). \tag{2}$$

Here, $z(t) = \mathbf{col}\left\{z(t,i)|_{i=1}^N\right\}$ and $v(t) = \mathbf{col}\left\{v(t,i)|_{i=1}^N\right\}$. Moreover, t and i stand respectively for the temporal variable and the index number of a subsystem, $x(t,i)$ represents the state vector of the ith subsystem $\mathbf{\Sigma}_i$ at time t, $z(t,i)$ and $v(t,i)$ respectively its output vector to other subsystems and input vector from other subsystems, $y(t,i)$, $d(t,i)$ and $w(t,i)$ respectively its output vector, input/process disturbance vector and measurement error vector. To distinguish the output vector $z(t,i)$ and the input vector $v(t,i)$ respectively from the output vector $y(t,i)$ and the input vector $d(t,i)$, $z(t,i)$ and $v(t,i)$ are called internal output/input vectors, while $y(t,i)$ and $d(t,i)$ external output/input vectors.

Throughout this paper, it is assumed that the dimensions of the vectors $x(t,i)$, $v(t,i)$, $d(t,i)$, $w(t,i)$, $z(t,i)$ and $y(t,i)$, are respectively $m_{\mathbf{T}i}$, $m_{\mathbf{S}i}$, $m_{\mathbf{d}i}$, $m_{\mathbf{w}i}$, $m_{\mathbf{z}i}$ and $m_{\mathbf{y}i}$. From these assumptions and Eq. (2), the dimension of the SCM Φ is clearly $\sum_{i=1}^N m_{\mathbf{S}i} \times \sum_{i=1}^N m_{\mathbf{z}i}$.

In the above description, every plant subsystem is permitted to have different dynamics, but their dynamics are required to be time invariant. On the other hand, the relation of Eq. (2) reflects the fact that the internal inputs of a subsystem are actually constituted from and only from some internal outputs of other subsystems. As only time invariant systems are investigated, the SCM Φ is assumed to be a constant matrix. Under such a situation, it can be assumed that every row of the matrix Φ has only one nonzero element which is equal to one. This assumption does not introduce any restrictions on the structure of the whole system. In fact, it still permits the above model to describe a system in which different internal outputs of the same subsystem affect different subsystems, a system in which one internal output simultaneously affects several subsystems, as well as a system with some subsystem inputs dependent on the outputs of multiple subsystems. Note that large-scale systems are usually sparse. It appears safe to declare that compared to the number of plant states, the dimension of this matrix is in general much smaller (D'Andrea & Dullerud, 2003; Siljak, 1978; Zhou, 2013).

It is worthwhile to mention that almost all the results of this paper remain valid even if the above assumption on the SCM Φ is not satisfied. But this assumption may make controllability/observability verification computationally more efficient, which can be seen from the discussions after the observation verification algorithm in Section 3. Moreover, this assumption does not mean that influence strengths among all subsystems are equivalent. In fact, different subsystem influence strengths can be reflected in both the SCM and the subsystem parameter matrices like $A_{\mathbf{TS}}(i)$, $A_{\mathbf{SS}}(i)$, etc. (D'Andrea & Dullerud, 2003; Zhou, 2013). In this paper, in order to reduce computation complexity, influence strengths among subsystems are selected to be included in their parameter matrices.

The above description is a modification of the model originally suggested in D'Andrea and Dullerud (2003) for describing the dynamics of a spatially invariant plant and utilized in many other studies such as Zhou (2013). The differences are that the model of Eqs. (1) and (2) permits its subsystems to have different dynamics and connections among subsystems to be arbitrary. This makes it capable of describing dynamics of a larger class of physical systems. This representation is very similar to a plant SSM, but knowledge on subsystem connections is described more explicitly. To avoid confusions, it is sometimes called an SSMR (Zhou, 2013).