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Predictor based stabilization of neutral type systems with input delay[☆]

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ABSTRACT

In this contribution we present an extension of the prediction scheme proposed in Manitius and Olbrot (1979) for the compensation of the input delay to the case of linear neutral type systems with input delay. For simplicity of the presentation we treat the case of systems with one state delay.

1. Introduction

Time delay in the input/output signals appears in mathematical models of various technical and technological processes – rolling mills, communication networks, traffic systems, combustion processes, biological and chemical processes – just to mention some of them. The predictive scheme for the compensation of actuator delays known now as Smith predictor has been proposed in Smith (1959). Starting from this publication a lot of new prediction techniques to control systems with time-delay in the input and/or output signals have been developed, see Artstein (1982), Fiagbedzi and Pearson (1986), Krstic (2009), Krstic and Smyshlyaev (2008), Kwon and Pearson (1980), Manitius and Olbrot (1979), Mazenc et al. (2003), Michiels, Engelborghs, Vansevenant, and Roose (2002), Niculescu and Annaswamy (2003) and references therein. In comparison to the case of systems described by linear time-invariant ordinary differential equations with delay in the control input, very few results are available for systems with both input and state delays. This fact has been emphasized in Krstic (2009).

In this contribution we present an extension of the prediction scheme proposed in Manitius and Olbrot (1979) for the compensation of the input delay in the computation of stabilizing controllers for linear neutral type systems with input delay. The case of retarded type systems has been studied in Kharitonov (2014). For simplicity of the presentation we treat the case of systems with one state delay, but the obtained results can be extended to the case of systems with multiple state delays, as well.

In Section 2 we provide basic notations used in the contribution, and give a formal statement of the stabilization problem for neutral type systems with input delay. Section 3 is devoted to the fundamental matrix of a neutral type system. Here it is shown that both the matrix and its first time derivative admit upper bounds. At the end of the section an explicit expression for the solutions of the system is given. Then, in Section 4, we apply this expression for the computation of future states of the system in order to compensate the input delay. These future states are used in the construction of stabilizing control laws. It is shown that such a control law is described by an integral equation, similar to that obtained in Manitius and Olbrot (1979), with additional terms due to the presence of the state delay in the system. Section 5 is dedicated to the stability analysis of the closed-loop system. The principal goal of the section is an upper exponential estimate for the solutions of the closed-loop system. In the computation of this estimate we assume that a similar estimate is already available for the solutions of an auxiliary neutral type system. Having in mind to rid of the assumption we present a Lyapunov type stability analysis. As the system under investigation contains state delay we are not able to use in this analysis Lyapunov functions any more, and should address Lyapunov–Krasovskii functionals. To this end we introduce in Section 6 some basic results concerning functionals involved in such an analysis.

In Section 7 we start with a simple modification of the backstepping transformation of the control variable proposed in Krstic and Smyshlyaev (2008). This transformation allows to present the

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closed-loop system in a form more appropriate for the consequent stability analysis. For the transformed system we propose a Lyapunov functional, similar to that of Krstic (2009), with a single modification—an auxiliary quadratic functional is used instead of the quadratic Lyapunov form. As a result we first obtain an upper exponential estimate for the solutions of the transformed system, and then we derive a similar exponential estimate for the original control variable. Several examples, illustrating the computation of the stabilizing control laws, are given in Section 8.

2. Problem formulation

Given a time-delay system of the form

$$\frac{d}{dt}\left[x(t)-Dx(t-h)\right]=A_0x(t)+A_1x(t-h)+Bu(t-\tau), \qquad (1)$$

where A_0, A_1, D are $n\times n$ matrices, and B is $n\times m$ matrix. The system delays satisfy the inequalities $\tau\geq h>0$. We assume that an initial function $\varphi:[-h,0]\rightarrow R^n$ belongs to the space $C^1([-h,0],R^n)$ of continuously differentiable functions. Let $x(t,\varphi)$ stand for the solution of system (1) under the initial condition

$$x(\theta,\varphi)=\varphi(\theta),\quad \theta\in[-h,0],$$

and $x_t(\varphi)$ denotes the restriction of the solution to the segment $[t-h,t]$,

$$x_t(\varphi):\theta\rightarrow x(t+\theta,\varphi),\quad \theta\in[-h,0].$$

We omit argument φ in these notations, and write $x(t)$ and x_t instead of $x(t,\varphi)$ and $x_t(\varphi)$, when no confusion may arise.

The euclidean norm is used for vectors, and the induced matrix norm for matrices. The space $C^1([-h,0],R^n)$ is supplied with the uniform norm,

$$\|\varphi\|_h=\sup_{\theta\in[-h,0]}\|\varphi(\theta)\|.$$

In the following we assume that there exist matrices F_0 and F_1, such that the system

$$\frac{d}{dt}\left[x(t)-Dx(t-h)\right]=(A_0+BF_0)\,x(t)+(A_1+BF_1)\,x(t-h), \qquad (2)$$

is exponentially stable.

Problem. Find a control law under which the system (1) coincides with (2).

Remark 1. Exponential stability of system (2) implies that matrix D is Schur stable. In this case there exist $d\geq 1$ and $\rho\in(0,1)$, such that the following inequality

$$\left\|D^j\right\|\leq d\rho^j,\quad j=0,1,2,\ldots \qquad (3)$$

holds (see Theorem 4.5 in Leonov and Shumafov (2012)).

We start with a formal definition of what is known as a fundamental matrix of system (1), see Hale and Verduyn-Lunel (1993).

Definition 1. A square $n\times n$ matrix $K(t)$ is said to be a fundamental matrix of system (1) if it satisfies the following conditions:

(1) Initial condition: $K(t)=0_{n\times n}$, for $t<0$, and $K(0)=I$.
(2) For $t\geq 0$ the matrix is a solution of the equation

$$\frac{d}{dt}\left[K(t)-K(t-h)D\right]=K(t)A_0+K(t-h)A_1. \qquad (4)$$

(3) Sewing condition: The difference

$$K(t)-K(t-h)D \qquad (5)$$

is continuous for $t\geq 0$.

Then, we present an auxiliary statement.

Lemma 1. *Given a positive $\tau=lh+\delta$, where l is an entire number, and $\delta\in[0,h)$, there exist positive constants η_0 and η_1 that satisfy the inequalities*

$$\sup_{t\in[0,\tau]}\|K(t)\|\leq\eta_0, \qquad (6)$$

$$\sup_{t\in[0,\tau]/\{0,h,\ldots,lh\}}\left\|\frac{dK(t)}{dt}\right\|\leq\eta_1. \qquad (7)$$

Proof. See Appendix for the proof.

Let $K(t)$ be the fundamental matrix of system (1), and $x(t)$ is a solution of the system, then the following expression holds, see Hale and Verduyn-Lunel (1993)

$$\begin{aligned}x(t)=&\int_{t_0}^{t}K(t-\xi)Bu(\xi-\tau)d\xi\\&+\int_{-h}^{0}K(t-t_0-\theta-h)A_1x(t_0+\theta)d\theta\\&+\int_{-h}^{0}K'(t-t_0-\theta-h)Dx(t_0+\theta)d\theta\\&+K(t-t_0)[x(t_0)-Dx(t_0-h)],\quad t\geq t_0.\end{aligned}$$

Here $K'(t)$ stands for the first derivative of the matrix,

$$K'(t)=\frac{dK(t)}{dt}.$$

Remark 2. Computing the integral

$$\int_{-h}^{0}K'(t-t_0-\theta-h)Dx(t+\theta)d\theta$$

one has to remember that matrix $K(t)$ has a jump discontinuity at points multiple to h. Therefore, the derivative $K'(t-t_0-\theta-h)$ includes Dirac type generalized functions.

3. General scheme

Given a solution $x(t)$ of system (1), then

$$\begin{aligned}x(t+\tau)=&\int_{-\tau}^{0}K(-\xi)Bu(t+\xi)d\xi+K(\tau)\left[x(t)-Dx(t-h)\right]\\&+\int_{-h}^{0}K(\tau-\theta-h)A_1x(t+\theta)d\theta\\&+\int_{-h}^{0}K'(\tau-\theta-h)Dx(t+\theta)d\theta,\end{aligned} \qquad (8)$$

and

$$\begin{aligned}x(t+\tau-h)=&\int_{-\tau}^{-h}K(-h-\xi)Bu(t+\xi)d\xi\\&+K(\tau-h)\left[x(t)-Dx(t-h)\right]\\&+\int_{-h}^{0}K(\tau-\theta-2h)A_1x(t+\theta)d\theta\\&+\int_{-h}^{0}K'(\tau-\theta-2h)Dx(t+\theta)d\theta.\end{aligned} \qquad (9)$$

We start with a control law of the form

$$u(t)=F_0x(t+\tau)+F_1x(t+\tau-h),\quad t\geq 0, \qquad (10)$$

where $x(t+\tau)$ and $x(t+\tau-h)$ on the right-hand side of the preceding expression are replaced by (8) and (9), respectively. As a result we arrive at a control law of the form

$$u(t)=f(u_t,x_t),\quad t\geq 0,$$

where

$$u_t:\xi\rightarrow u(t+\xi),\quad \xi\in[-\tau,0],$$