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Junli Liang, Yuanyuan Chen, H. C. So, *Fellow, IEEE*, and Yang Jing

Abstract

Localization of a target of interest based on measurements using multiple spatially separated sensors has been one of the central problems in numerous application areas. Many measurement types in the three-dimensional space are convertible to distance information which can be classified as range, range-difference and range-sum for different applications, which correspond to solving a set of circular, hyperbolic and elliptic equations, respectively. In this work, we develop methods for circular/hyperbolic/elliptic localization. The key idea is to remove the Euclidean norm in the resultant optimization formulations via the introduction of auxiliary vector magnitude and phase variables. Moreover, the vector magnitude and phase angle parameters are decoupled, leading to separable and computationally efficient updating rules. The optimality of the developed methods is demonstrated by comparing with the Cramér-Rao lower bound via computer simulations.

Keywords: target localization, range, range-difference, range-sum.

1. Introduction

Target localization using multiple spatially separated sensors with *a priori* known positions has been an important problem in radar, sonar, global positioning system, wireless communications, multimedia and sensor network [1]–[3]. Representative applications include emergency assistance, intelligent transportation, location based advertising, speaker tracking and internet of things. Usually, localization systems collect the sensor measurements containing the distance and/or bearing information from which the target position can be computed. In this paper, we focus on algorithm development for distance based positioning.

In general, the distance information refers to range [4], range-difference [5] or range-sum [6]. The range is the distance from the target to a sensor. In a time-synchronized system, it can be estimated by measuring the time-of-arrival (TOA) [7], namely, the one-way propagation time of the signal traveling between them, and then multiplying by the known propagation speed. On the other hand, for received signal strength (RSS) based positioning [8], average power attenuation is utilized to determine the range according to a signal path-loss model. Under the ideal case of zero estimation error and considering two-dimensional (2-D) localization, each range corresponds to a circle centered at the sensor where the target must be a point on the circumference, implying that at least 2 sensors are needed to calculate the position. The range-difference is the difference between two ranges which can be obtained by estimating the time-difference-of-arrival (TDOA) [9] between a pair of sensor outputs or differential RSS (DRSS) [10] in time or RSS based systems, respectively. Note that comparing with the TOA and RSS, TDOA does not require clock synchronization in the target while there is no need to know the transmit signal power in DRSS. In 2-D geometry, each noise-free range-difference measurement defines a hyperbola on which the source must lie and the target location is given by intersection of at least two hyperbolae. Finally, range-sum arises in multiple-input multiple-output (MIMO) radar [11] and multistatic sonar [12] systems consisting of two kinds of sensors, namely, transmitters and receivers. Here, we can measure the sum of two TOAs, that is, propagation time from a transmitter to the target and that from the target to a receiver, and multiplying it by the propagation speed for its determination. Each range-sum produces an ellipse in the 2-D space and intersecting at least two ellipses yields the target location. It is clear that each range, range-difference and range-sum define a sphere, hyperboloid and ellipsoid, respectively, in the 3-D case. Nevertheless, locating the target is not a trivial task because these measurements have nonlinear relationships with the position, as indicated in the circular, hyperbolic and elliptic equations.

Generally speaking, there are two localization categories given the range, range-difference and range-sum measurements. The first deals with the corresponding circular, hyperbolic or elliptic equations explicitly to calculate the location. This can be straightforwardly formulated as an unconstrained nonlinear optimization problem or converted to an equivalent constrained form. Due to the nonlinear nature, it is not guaranteed to obtain the globally optimal solutions. The standard solvers for the former are the nonlinear least squares (NLS) and maximum likelihood (ML) estimators [11], [13]–[14]. For the latter, the constrained nonlinear optimization problem can be relaxed as a convex program and solved via semidefinite relaxation (SDR) [15], and localization algorithms using range, range-difference and range-sum measurements have been presented in [5], [16]–[17] and [18], respectively. Recently, the Lagrange programming neural network (LPNN) [19] approach, which is an analog computational technique based on the Lagrange multiplier theory, is successfully applied in these constrained formulations

J. Liang (Corresponding author), Y. Chen, and Y. Jing are with School of Electronics and Information, Northwestern Polytechnical University, Xi’an, China (e-mail: liangjunli@nwpu.edu.cn). H. C. So is with Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China (e-mail: hcso@ee.cityu.edu.hk).