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Structural analysis of sensor location for disturbance rejection by measurement feedback[☆]

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ABSTRACT

The Disturbance Rejection by Measurement Feedback (DRMF) is a well known problem mixing control and estimation aspect, whose resolution relies on a good knowledge of the system structural properties. The solvability conditions are highly dependent on the sensor locations. In this paper we analyze the sensor location issues for the DRMF of structured systems which are a large class of parameter dependent linear systems. The sensor location for this problem is already solved in the literature for the case of systems with a single disturbance. It turns out that the sensors must measure state variables in regions close enough to the action of the disturbance. In the multiple disturbance case, the problem is much more complex; some close measurements may be useless while others more distant are useful. In this paper we solve the multiple disturbance case and provide with a full characterization of the sensor location for DRMF. Sets of state variables that are not useful to measure because they never belong to a minimal sensor solution as well as sets of state variables that belong to minimal solutions are determined easily on the system associated graph.

1. Introduction

This paper is concerned with linear systems which are affected by unmeasurable disturbances and we look for exact disturbance rejection (*i.e.* a zero disturbance-controlled output transfer matrix) by measurement feedback. The problem of disturbance rejection by state feedback is a very well known problem and was one of the emblematic problems of the geometric theory, see Basile and Marro (1992) and Wonham (1985). In the case where the state is not available for measurement, the problem of disturbance rejection by measurement feedback is more complex and has been solved in an elegant way in geometric terms, see Schumacher (1980) and Willems and Commault (1981). The characterization of the solutions with internal stability and the set of fixed modes have been also characterized geometrically, see del Muro Cuellar and Malabre (2001) and Eldem and Ozguler (1988). In the framework of structured systems introduced by Lin (1974), the DRMF problem has been solved graphically in Commault, Dion, and Hovelaque (1997) and van der Woude (1993). It turns out that the problem is generically solvable if and only if a simple graphical condition is satisfied. The case of structured transfer matrix systems has been considered in van der Woude (1996). Other approaches allow to minimize some norm of the disturbance to controlled output transfer matrix, see for example Vidyasagar (1987).

The sensor location problem, *i.e.* how many sensors do we need and where should they be located, has received significant attention during the last decade. This problem has already been studied in different contexts for several problems. For example the placement of sensors for feedback control was studied using balanced realizations or minimizing closed loop performance metrics, see Balas and Young (1999) and van de Wal and de Jager (2001) for a survey of input/output selection. A disturbance rejection goal has been introduced by Lim (1997) in the context of sensor and actuator placement for flexible structure applications. The sensor location problem has been also studied for fault tolerance and diagnosis see Blanke, Kinnaert, Lunze, and Staroswiecki (2003), Krysander and Frisk (2008) and Staroswiecki, Hoblos, and Aitouche (2004). In the context of structured systems, sensor location for observability was considered by Boukhobza and Hamelin (2009), Commault, Dion, and Trinh (2008) and Staroswiecki et al. (2004). The sensor location for the DRMF problem has been studied in de Oliveira and Geromel (2000) in the context of robust design minimizing the H_2 norm of the disturbance-regulated output transfer matrix. Few results have been published on the structural analysis of sensor placement for the DRMF problem.

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Commault, Dion, and Do (2011) proposed a new necessary and sufficient condition for the DRMF problem which is well adapted to tackle sensor location problems. They gave the minimum number of required sensors for DRMF and solved the sensor location problem for the case of a unique disturbance. It is shown that the measurements must be taken close enough to where the disturbance acts. In the case of multiple disturbances, only partial results are available, it is shown in Commault et al. (2011) that measuring outside a given subset is useless.

In this paper we provide with a full characterization of sensor location for the DRMF problem with a minimum number of sensors, in the case of multiple disturbances. We determine with only the knowledge of the system structure, independently of the parameters value, sets of state variables that are of interest to measure, *i.e.* which belong to minimal solutions and other sets of state variables which are useless. It is shown that there exist two types of useless measurements, those which arrive too late to inform on the disturbance or those which provide with an insufficient information. These sets are determined easily from system invariants as essential orders and using separators on the system associated graph.

The paper is structured as follows. We formulate the problem of sensor location for disturbance rejection by measurement feedback in the framework of structured systems in Section 2. Known results on the problem are recalled in Section 3. Section 4 presents the specific features of the multiple disturbance case and gives the main results which characterize the sets of state variables that are of interest for the problem. Section 5 gives a structural overview of the sensor location for DRMF together with a simple pedagogical example and presents some computational aspects. Some concluding remarks are given in Section 6.

2. Problem formulation

2.1. Sensor location for disturbance rejection by measurement feedback

Let us consider the linear structured system Σ_Λ (Lin, 1974) given by:

$$\Sigma_\Lambda \begin{cases} \dot{x}(t) = A_\Lambda x(t) + B_\Lambda u(t) + E_\Lambda d(t) \\ y(t) = C_\Lambda x(t) \\ z(t) = H_\Lambda x(t), \end{cases} \tag{1}$$

where $u(t) \in \mathbb{R}^m$ is the control input, $d(t) \in \mathbb{R}^q$ is the unmeasurable disturbance, $x(t) \in \mathbb{R}^n$ is the state, $y(t) \in \mathbb{R}^p$ is the regulated output and $z(t) \in \mathbb{R}^\nu$ the measured output provided by a sensor network. This system is called a linear structured system if the entries of the composite matrix $J_\Lambda = \begin{bmatrix} A_\Lambda & B_\Lambda & E_\Lambda \\ C_\Lambda & 0 & 0 \\ H_\Lambda & 0 & 0 \end{bmatrix}$ are either fixed zeros or independent parameters (not related by algebraic equations). $\Lambda = \{\lambda_1, \lambda_2, \ldots, \lambda_N\}$ denotes the set of independent parameters of the composite matrix J_Λ.

For such systems, one can study generic properties, *i.e.* properties which are true for almost all values of the parameters collected in Λ, Murota (1987). More precisely, a property is said to be generic (or structural) if it is true for all values of the parameters outside a proper algebraic variety of the parameter space.

The problem of Disturbance Rejection by Measurement Feedback (DRMF), amounts to find (if possible) a parameter dependent dynamic measured output feedback compensator

$$\Sigma_{zu} \begin{cases} \dot{w}(t) = Lw(t) + Rz(t) \\ u(t) = Sw(t) + Pz(t), \end{cases} \tag{2}$$

such that in the closed loop system, the unknown disturbance $d(t)$ has no effect on the controlled output $y(t)$, see Fig. 1.

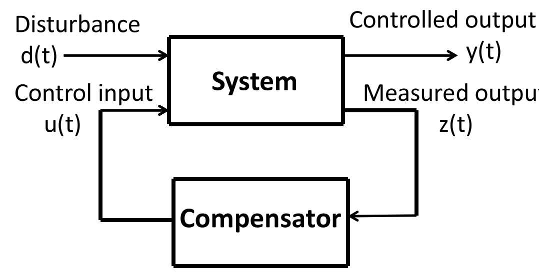

Fig. 1. Dynamic feedback compensation.

More precisely, in transfer matrix terms, we look for a dynamic compensator $u(s) = F(s)z(s)$, where $F(s)$ is a proper rational matrix, such that the closed loop system transfer matrix from disturbance $d(s)$ to the controlled output $y(s)$ is identically zero. When such a compensator exists, the sensor set determined by H_Λ in (1) is called a DRMF solution.

The sensor location problem amounts to find the minimal number of sensors and the associated set of state variables to be measured for solving the considered problem.

In this paper we analyze the sensor location issues for the DRMF of structured systems. We assume that all states can be measured individually by a sensor and we focus on minimal solutions (*i.e.* solutions with the minimal number of sensors).

The formulation of the sensor location problem for the DRMF is then as follows:

Given the matrices $A_\Lambda, B_\Lambda, C_\Lambda, E_\Lambda$ of system (1), find (if possible) a matrix H_Λ (representative of sensors measuring individual states) which ensures a minimal DRMF solution. Then, among all possible sensors, characterize those that are useful *(which belong to at least one minimal DRMF solution), those that are* useless *(which never belong to a minimal DRMF solution) and those that are* essential *(which should be measured because they are part of any minimal DRMF solution).*

2.2. Graph representation and basic tools

A directed graph $G(\Sigma_\Lambda) = (V, W)$ can be associated with the structured system Σ_Λ of type (1):

- the vertex set is $V = U \cup D \cup X \cup Y \cup Z$ where U, D, X, Y and Z are the input, disturbance, state, regulated output and measured output sets given by $\{u_1, u_2, \ldots, u_m\}$, $\{d_1, d_2, \ldots, d_q\}$, $\{x_1, x_2, \ldots, x_n\}$, $\{y_1, y_2, \ldots, y_p\}$ and $\{z_1, z_2, \ldots, z_\nu\}$, respectively,
- the arc set is $W = \{(u_i, x_j) | B_{\Lambda,ji} \neq 0\} \cup \{(d_i, x_j) | E_{\Lambda,ji} \neq 0\} \cup \{(x_i, x_j) | A_{\Lambda,ji} \neq 0\} \cup \{(x_i, y_j) | C_{\Lambda,ji} \neq 0\} \cup \{(x_i, z_j) | H_{\Lambda,ji} \neq 0\}$, where $A_{\Lambda,ji}$ (*resp.* $B_{\Lambda,ji}, E_{\Lambda,ji}, C_{\Lambda,ji}, H_{\Lambda,ji}$) denotes the entry (j, i) of the matrix A_Λ (*resp.* $B_\Lambda, E_\Lambda, C_\Lambda, H_\Lambda$).

Let V_1, V_2 be two nonempty subsets of the vertex set V of the graph $G(\Sigma_\Lambda)$. There exists a *path* from V_1 to V_2 ($V_1 - V_2$ path) if there are vertices $i_0, i_1, \ldots, i_\nu$ such that $i_0 \in V_1, i_\nu \in V_2, i_t \in V$ for $t = 0, 1, \ldots, \nu$ and $(i_{t-1}, i_t) \in W$ for $t = 1, 2, \ldots, \nu$. The path is called *simple* if every vertex on the path occurs only once. The length of a path is the number of its arcs. A *shortest path* from i_0 to i_ν is a simple path between these two vertices such that the number of its arcs is minimum.

Two paths from V_1 to V_2 are said to be *disjoint* if they consist of disjoint sets of vertices. r paths from V_1 to V_2 are said to be disjoint if they are mutually disjoint, *i.e.* each two of them are disjoint. A set of r disjoint and simple paths from V_1 to V_2 is called a *linking from V_1 to V_2* (*$V_1 - V_2$ linking*) of size r. A linking consisting of a maximal number of disjoint paths from V_1 to V_2 is called a *maximal $V_1 - V_2$ linking* and its size is denoted $\rho(V_1 - V_2)$. The set of $V_1 - V_2$