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Estimation of chirp signals with time-varying amplitudes☆

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ABSTRACT

The problem of parameters estimation of signals composed of an unknown number of chirps with time-varying amplitude is presented using a sparse reconstruction framework. The method employs a parametric model using a weighted combination of splines to model the time-varying nature of the signal amplitudes. To obtain high-resolution of the frequencies and to avoid large dimensional matrices, a dictionary refinement technique is employed. The method can accurately estimate the amplitude and frequency parameters of multiple signal components, and may be extended to allow for non-linear chirps. Furthermore, an efficient implementation to solve the resulting optimization problem is proposed. Results on both synthetic and experimental signals illustrate the efficient performance of the algorithm.

1. Introduction

Signals with time varying frequency components, chirps, are common in many real-world applications, ranging from animal vocalization and human speech to music signals, and are widely used in many fields of signal processing, with applications in, for instance, acoustic scattering, mechanical vibration, geophysics, sonar, radar, and telecommunication (see, for instance, [1,2]). The linear chirp signal, also known as a linear frequency modulated signal (LFM), has a fundamental role in sonar and radar target detection, localization, and classification, as it may provide excellent range resolution and Doppler invariance [3,4].

Due to the importance of such signals, notable attention has been given to develop efficient estimation algorithms for them. Most of these works assume that the amplitude of the linear chirp signals are constant or normalized. Such signals are characterized by the phase function, the instantaneous frequency, which is a function of the starting frequency and the frequency rate. Some of the methods described in the literature exploit phase unwrapping [5], maximum likelihood formulation [2,6], least squares minimization [7], sample covariance matrix estimation techniques [8], as well as integrated cubic phase function estimators [9]. In addition, time-frequency analysis has been proven to be a powerful and effective tool to analyze chirp signals. Direct ways to obtain the parameters from such representations include extraction of the peak from the time-frequency plane and adaptive estimation methods, as reviewed in [10], as well as techniques such as ant colony optimization [11]. Other alternative ways include parameterizing the time-frequency plane using methods like the Radon or the Hough transforms. Such techniques can be developed by combining them with a time-frequency analysis tool, such as the Radon-ambiguity transform [12], or the Huang–Hough transform [13]. These methods rotate or warp the time-frequency plane to form a new parametric domain. A similar approach is exploited by the fractional Fourier transform (FRFT), which transforms the signal from the time-domain to a generalized frequency-domain. It can be interpreted as a signal decomposition in terms of a chirp basis, which has a notable potential for analyzing chirp signals [14,15].

However, in some applications, the signals might have time-varying amplitudes by design (e.g. AM-FM signals in communication or speech coding) or as a result from signal fluctuations while propagating. This occurs, for instance, in active sonar, wherein a chirp signal is transmitted towards a potential target. This results in the hydrophone receiving echoes containing multiple components produced by rigid reflections and elastic scattering, which have different characteristics varying with frequency. These reflections may then be used to gain insight of the nature of the target. Due to the time-varying nature of such reflections, an estimator assuming a constant amplitude reflection will typically suffer notable bias. However, some featured approaches for estimation of time-

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varying amplitude chirp signals can be extended from those of sinusoidal and polynomial phase signals. The parameters of sinusoidal signals can be obtained by the least-squares estimators [16], the nonlinear least-squares (NLS) approach [17], with the latter one also working for polynomial phase signals. Some other available methods include the maximum likelihood estimator [18] and the cyclic moments-based approaches [19]. A noteworthy approach was presented in [20], where the amplitude and phase of a component was expressed by a weighted sum of some arbitrary sets, and the parameters were computed through the maximum likelihood estimation of these weights. The NLS approach can be used on a mono-component chirp signal with randomly time-varying amplitude, and may also be combined with the high-order ambiguity function [21]. The iterative NLS estimators presented in [22] can also solve the estimation problem of harmonic chirp signals. Besides, by using the parameterized demodulation method, the wide-band signals can be transformed to narrow-band ones, which may then be extracted using various decomposition methods [23,24]. The decomposition problem may also be solved using a linear system where the instantaneous frequencies and instantaneous amplitudes are modeled as Fourier series [25]. One may also use time-frequency analysis to estimate the instantaneous frequencies and the power of each component without suffering from cross-terms by using methods such as the S-method [26]. Moreover, a method based on FRFT was presented in [27], which can estimate the parameters of multiple component signals, although the amplitude was there assumed to be linearly modulated.

Herein, we investigate parameter estimation of a signal containing multiple chirp signals with time-varying amplitudes embedded in Gaussian noise using a sparse reconstruction framework. Some related work have been done to estimate chirp signals with constant amplitude. In [28], a Gabor dictionary was used to estimate multicomponent chirp signals using the matching pursuit algorithm, in combination with the Hough transform. The drawback of such an approach is that it only allows for the estimation of the frequency rate. To allow for the estimation of an unknown number of chirp signals, a LASSO-based framework was presented in [29]. Using this algorithm, the starting frequency and the frequency rate of every component can be found simultaneously. Since the estimation accuracy depends on the grid structure of the dictionary, an alternative approach was proposed in [30], wherein an iterative framework over two dictionaries was used, each one being defined over a distinct parameter. A higher resolution estimate could then be obtained by adding an NLS search after the LASSO procedure [31]. In this paper, this framework is extended to allow for components with a time-varying amplitude, using an idea reminiscent of the one proposed in [20,32]. Herein, we use a low order spline basis with uniformly placed knots [33] to capture the time-varying nature of the amplitudes. A similar approach has recently been used for estimation of amplitude modulated sinusoids in [34] and for chroma estimation in [35]. By introducing a spline basis to represent the time-varying nature of the amplitudes, the signal can be characterized by the corresponding weighting coefficients. These are then utilized to estimate the parameters capturing the behavior of the signal. An efficient implementation to solve the resulting optimization problem is presented based on the alternating direction method of multipliers (ADMM) framework [36].

The paper is organized as follows: in the next section, we introduce the considered signal model and the proposed estimation algorithm. Then, in Section 3, the efficient implementation of this estimator is introduced. Section 4 examines the performance of the proposed algorithm. Finally, Section 5 contains our conclusions.

2. Signal model and the proposed estimator

2.1. Parametric signal model

Consider the signal

$$y(t_n) = \sum_{k=1}^{K} a_k(t_n) e^{j\phi_k(t_n)} + e(t_n) \tag{1}$$

for $n = 1, 2, \cdots, N$, where N denotes the number of available samples, K the (unknown) number of components, $a_k(t_n)$ and $\phi_k(t_n)$ are the time dependent amplitude and phase functions of the kth component, respectively, whereas $e(t_n)$ is an additive noise term, here assumed to be well modeled as being a white and zero mean complex Gaussian random process. The instantaneous frequency, which is defined as $f_k(t) = \frac{1}{2\pi} \mathrm{d}\phi_k(t)/\mathrm{d}t$, is assumed to be linearly modulated, or to be well approximated as being linear within short time intervals. Thus, the phase function and the instantaneous frequency may be expressed as

$$\phi_k(t_n) = 2\pi f_k^0 t_n + \pi r_k t_n^2 \tag{2}$$

and

$$f_k(t_n) = f_k^0 + r_k t_n \tag{3}$$

respectively, where f_k^0 denotes the starting frequency, and r_k the frequency rate of the kth component. In this work, we assume that the complex-valued amplitude varies slowly during the observation period, with the highest frequency of the modulation being much lower than the frequency range of the signal. This form of signals can be often found in active sonar applications, where the reflected signals differ from the transmitted signal due to the nature of the backscattering. Similarly, the model may be used to detail time-varying audio signals (see, e.g., [37,38]) or AM-FM signals. In the former case, the used model would be appropriate to detail the discrete-time analytical representation of the signal, often used in such applications to form a more compact signal model, whereas for the latter, it would instead be used for the demodulated signal. The considered problem here is that of estimating the number of signals, K, as well as time-varying functions $a_k(t_n)$ and $f_k(t_n)$, for each component.

To model the amplitudes' time-varying nature, a combination of spline basis functions (as defined in Section 5.1 in [33]) may be used, such that the amplitudes are detailed as

$$a_k(t_n) = \sum_{r=1}^{R} \gamma_r(t_n) s_{r,k} \tag{4}$$

To ensure the validity of the representation, we assume that the components have no overlap in frequency. For the kth component, the amplitude may then be expressed as

$$\mathbf{a}_k = \mathbf{\Gamma} \mathbf{s}_k \tag{5}$$

where

$$\mathbf{a}_k = \begin{bmatrix} a_k(t_1) & a_k(t_2) & \cdots & a_k(t_N) \end{bmatrix}^T \tag{6}$$

where $(\cdot)^T$ denotes the transpose, and with

$$\mathbf{s}_k = \begin{bmatrix} s_{1,k} & s_{2,k} & \cdots & s_{R,k} \end{bmatrix}^T \tag{7}$$

denoting the weighting coefficient, whereas

$$\mathbf{\Gamma} = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_R \end{bmatrix} \tag{8}$$

is the designed spline matrix, with

$$\gamma_r = \begin{bmatrix} \gamma_r(t_1) & \gamma_r(t_2) & \cdots & \gamma_r(t_N) \end{bmatrix}^T \tag{9}$$

for $r = 1, 2, \cdots, R$, being the spline basis. With $z_k(t_n) = e^{j\phi_k(t_n)}$, the signal model may then be expressed as

$$y(t_n) = \sum_{k=1}^{K} a_k(t_n) z_k(t_n) + e(t_n) \tag{10}$$