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Brief paper

Leader–follower consensus of time-varying nonlinear multi-agent systems[☆]

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ABSTRACT

A distributed consensus protocol is developed for a class of homogeneous time-varying nonlinear multi-agent systems in this paper. The agents dynamics are supposed to be in the strict feedback form and satisfy Lipschitz conditions with time-varying gains. It is firstly shown that the leader–follower consensus problem is equivalent to a conventional stabilization control problem of augmented high-dimensional multi-variable systems. By introducing an appropriate state transformation, the stabilization control problem is then converted into a design problem of an appropriate time-varying parameter, which plays a key role to cope with the time-varying nonlinear terms in the concerned systems. It is proved that the exponential consensus of the multi-agent system can be achieved with the proposed consensus protocol. A simulation example is given to illustrate the effectiveness of the proposed consensus protocol.

1. Introduction

Multi-agent systems have received much attention in the past couple of decades. This is mainly due to their potential wide application in many industrial and military areas, (Hu, Su, & Lam, 2013; Ren, Beard, & Atkins, 2007). In multi-agent systems, control protocols are normally designed for each agent based on local information obtained from its neighbors. The consensus problem is one of the most important problems in the study of multi-agent systems. It was investigated for single-integrator multi-agent systems in Fan, Feng, Wang, and Song (2013), Munz, Papachristodoulou, and Allgower (2011) and Ren et al. (2007), and for double-integrator multi-agent systems in Abdessameud and Tayebi (2013) and Dong and Huang (2013), respectively. The same problem was investigated for high-order-integrator multi-agent systems in He and Cao (2011) and Yu, Chen, Ren, Kurths, and Zheng (2011), and for general linear multi-agent systems in Seo, Shim, and Back (2009), Trentelman, Takaba, and Monshizadeh (2013) and Zhu, Jiang, and Feng (2014), respectively. The problem of leader selection in noisy networks was studied in Clark, Bushnell, and Poovendran (2014), Fitch and Leonard (2013), Hu and Feng (2010) and Lin, Fardad, and Jovanovic (2011).

It should be pointed out that almost all physical systems are nonlinear in nature. Recently some interesting research results have been obtained for nonlinear multi-agent systems. The consensus problem was studied for second-order nonlinear multi-agent systems in Hu et al. (2013), and for high-order nonlinear multi-agent systems in Li, Ren, Liu, and Fu (2011) and Wen, Duan, Yu, and Chen (2013), respectively. It should also be noted that strict feedback nonlinear systems, with high-order-integrator systems as their special case, represent many physical systems, see Krstic, Kokotovic, and Kanellakopoulos (1995). For this class of nonlinear multi-agent systems, the tracking problem was considered in Yoo (2013), while the consensus problem was considered in Wang and Ji (2012). All nonlinear dynamics in Hu et al. (2013), Li et al. (2011), Wang and Ji (2012) and Wen et al. (2013) are assumed to satisfy Lipschitz conditions with constant gains.

It can be observed from the aforementioned works that multi-agent systems are assumed to be time-invariant. However, many practical engineering systems are indeed time-varying. Stability analysis and stabilization of time-varying systems have been always an important topic in the area of systems and control, see for example, Alessandri and Rossi (2013), Du, Qian, Yang, and Li (2013), Ngoc (2014), and Yang and Huang (2012). With the same argument, many multi-agent systems would have time-varying

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dynamics. Unfortunately, there is little study on time-varying multi-agent systems. In fact, to our best knowledge, the consensus problem has not been investigated for time-varying nonlinear multi-agent systems in the strict feedback form, which motivates this study.

In this paper, we will study the leader–follower consensus problem of time-varying nonlinear multi-agent systems based on the so-called dynamic gain control approach, with which the control design problem can be converted into the design problem of some dynamic parameters, Zhang, Baron, Liu, and Boukas (2011) Zhang, Feng, and Sun (2012), and Zhang, Liu, Baron, and Boukas (2011). It will be shown that the exponential consensus of the time-varying nonlinear multi-agent system can be achieved with the proposed consensus protocol. The rest of the paper is organized as follows. Section 2 is devoted to introduction of preliminaries and problem formulation. Section 3 presents the design and analysis of the consensus protocol. A numerical example is given in Section 4 to illustrate the effectiveness of the proposed protocol which is followed by some conclusions in Section 5.

2. Preliminaries and problem formulation

In this section, we present some preliminaries and the problem formulation.

Denote by $\mathbb{R}$ the field of real numbers and $\mathbb{R}^{m\times n}$ the set of $m\times n$ real matrices. Let $\|\cdot\|$ denote the Euclidean norm for vectors, or the induced Euclidean norm for matrices. I_m is used to represent an identity matrix of m dimension. $A_1\otimes A_2$ denotes the Kronecker product of matrices A_1 and A_2. For a symmetric matrix Q, $\lambda_{\max}(Q)$ and $\lambda_{\min}(Q)$ denote the largest and the smallest eigenvalue of Q, respectively. The argument of functions will be omitted or simplified whenever no confusion can arise from the context. For example, we may denote $x_i(t)$ by x_i.

2.1. Graph theory

Some basic concepts and results from algebraic graph theory are introduced here. Suppose that a team consists of N agents. We use a weighted undirected graph $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ to model the interaction among these agents, where $\mathcal{V}=\{1,2,\ldots,N\}$ is the node set, $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$ is the edge set, and $\mathcal{A}=(a_{ij})\in\mathbb{R}^{N\times N}$ is the weighted adjacency matrix. Two agents are called adjacent, if there exists an edge between them. A graph is simple if there is no self-loops or repeated edges. An edge (i,j) in $\mathcal{G}$ denotes that agents i and j can obtain information from each other. The weighted adjacency matrix $\mathcal{A}$ associated with $\mathcal{G}$ is defined such that $a_{ij}>0$ if $(j,i)\in\mathcal{E}$, and $a_{ij}=0$ otherwise. Note that here $a_{ij}=a_{ji}, \forall i\neq j$, since $(j,i)\in\mathcal{E}$ implies $(i,j)\in\mathcal{E}$. A path on $\mathcal{G}$ between i_1 and i_l is a sequence of edges of the form (i_k,i_{k+1}), $k=1,2,\ldots,l-1$. If there exists a path between any two agents of $\mathcal{G}$, then $\mathcal{G}$ is said to be connected. The degree matrix of $\mathcal{G}$ is a diagonal matrix $\mathcal{D}=\operatorname{diag}(d_1,d_2,\ldots,d_N)$, where $d_i=\sum_{j=1}^N a_{ij}$ for $i=1,2,\ldots,N$. The Laplacian matrix of $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$, which is symmetric. A subgraph $\mathcal{H}$ of $\mathcal{G}$ is an induced subgraph if two agents of $\mathcal{H}$ are adjacent in $\mathcal{H}$ if and only if they are adjacent in $\mathcal{G}$. An induced subgraph $\mathcal{H}$ of $\mathcal{G}$ is called a component of $\mathcal{G}$ if it is maximal, subjected to be connected, see Wang and Ji (2012).

Consider a graph $\mathcal{G}$ associated with the system consisting of N agents and a leader. Regarding the N agents as the agents in $\mathcal{V}$, the relationships between agents can be described by a simple and undirected graph $\mathcal{G}$. $\bar{\mathcal{G}}$ contains $\mathcal{G}$ and a leader with edges between some agents and leader. The agents that are connected to the leader can receive the information from the leader agent. It is assumed that all the followers know the input of the leader agent, as in Wang and Ji (2012), or the input of the leader agent is zero, as in Dong and Huang (2013), Li et al. (2011) and Su and Huang (2012), and the leader agent receives no information from any follower agent. The connection weight matrix is denoted by $\mathcal{B}=\operatorname{diag}(b_1,b_2,\ldots,b_N)$, and $b_i\geq 0$. $b_i>0$ if and only if agent i is connected to the leader. $\bar{\mathcal{G}}$ is connected if at least one agent in each component is connected with the leader.

2.2. Problem formulation

Note that many consensus synthesis approaches are based on state information of the neighbors of each agent which is assumed to be available. However, in practical applications, not all state information is easily available. Instead only some of state information in terms of outputs is available. Therefore we will study the consensus problem based on measurable outputs.

Consider the following multi-agent system described by

$$\begin{cases}\dot{x}_{k,i} &= x_{k,i+1}+f_i\left(t,\bar{x}_{k,i}\right), \quad i=1,2,\ldots,n-1\\ \dot{x}_{k,n} &= u_k+f_n\left(t,\bar{x}_{k,n}\right),\\ y_k &= x_{k,1}, \quad k=0,1,\ldots,N\end{cases} \tag{1}$$

where $\bar{x}_{k,i}=\left(x_{k,1},x_{k,2},\ldots,x_{k,i}\right)$, $x_k=\bar{x}_{k,n}^{\mathrm{T}}$, $u_k\in\mathbb{R}$ and $y_k\in\mathbb{R}$ are the state, input and output of agent k, respectively. In our formulation, the agent indexed by 0 is referred as the leader and agents indexed by $1,2,\ldots,N$, are called the followers. We assume that the functions $f_i(\cdot)$, $i=1,2,\ldots,n$, in system (1), satisfy the following Lipschitz condition.

Assumption 1. For $i=1,2,\ldots,n$, and any $\left(t,\bar{x}_{k,i}\right)$, $\left(t,\bar{x}_{l,i}\right)\in\mathbb{R}^+\times\mathbb{R}^i$, the following inequality hold:

$$\left|f_i\left(t,\bar{x}_{k,i}\right)-f_i\left(t,\bar{x}_{l,i}\right)\right|\leq\eta(t)\sum_{j=1}^{i}\left|x_{k,j}-x_{l,j}\right| \tag{2}$$

where $\eta(t)=c_1e^{c_2t}$, with c_1 and c_2 being known nonnegative constants.

Remark 1. As shown in Mazenc, Praly, and Dayawansa (1994), for some systems with growth nonlinearities with respect to unmeasured state components, the problem of global output feedback stabilization may not be solvable. So some growth conditions are necessary in dealing with nonlinearities depending on unmeasured states. The nonlinear terms involved in many existing works (see Praly and Jiang (2004) and the references therein), admit an incremental rate depending only on the measured output, not depending on time t. System (1) satisfying Assumption 1 is indeed an intrinsic time-varying system, in the sense that $\eta(t)$ can be any known exponential function defined on $[0,+\infty)$. Many time-varying functions, such as, bounded functions including decreasing functions, polynomial functions, and logarithmic functions with respect to time t, can be bounded by appropriate exponential functions $c_1e^{c_2t}$ with c_1 and c_2 being known constants, and hence system (1) satisfying Assumption 1 includes many types of time-varying systems. It is noted that no research result was reported on the exponential stabilization problem of time-varying system (1) even with only one agent. The considered model is also more general than the most existing models, and in fact, it includes the following special cases.

(i) The systems considered in Wang and Ji (2012) satisfies (2) with $c_2=0$.

(ii) The systems considered in Su and Huang (2012) and Yoo (2013) are time-invariant systems described by (1) with $f_i(\cdot)=f_i\left(\bar{x}_{l,i}\right)$.

In this work, by choosing an appropriate time-varying parameter $L(t)$, we will design an output feedback consensus protocol for