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Brief paper

Feedback control of switched stochastic systems using randomly available active mode information☆

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ABSTRACT

Almost sure asymptotic stabilization of a discrete-time switched stochastic system is investigated. Information on the active operation mode of the switched system is assumed to be available for control purposes only at random time instants. We propose a stabilizing feedback control framework that utilizes the information obtained through mode observations. We first consider the case where stochastic properties of mode observation instants are fully known. We obtain sufficient asymptotic stabilization conditions for the closed-loop switched stochastic system under our proposed control law. We then explore the case where exact knowledge of the stochastic properties of mode observation instants is not available. We present a set of alternative stabilization conditions for this case. The results for both cases are predicated on the analysis of a sequence-valued process that encapsulates the stochastic nature of the evolution of active operation mode between mode observation instants. Finally, we demonstrate the efficacy of our results with numerical examples.

1. Introduction

The framework developed for switched stochastic systems provides accurate characterization of numerous complex real life processes from physics and engineering fields that are subject to randomly occurring incidents such as sudden environmental variations or sharp dynamical changes (Cassandras & Lygeros, 2006; Yin & Zhu, 2010). Stabilization problem for switched stochastic systems has been investigated in many studies (e.g., Costa, Fragoso, & Marques, 2004, de Farias, Geromel, do Val, & Costa, 2000, Fang & Loparo, 2002, Geromel, Goncalves, & Fioravanti, 2009, Ghaoui & Rami, 1996, Sathanantan, Adetona, Beane, & Keel, 2008 and the references therein).

Control frameworks developed for switched stochastic systems often require the availability of information on the active operation mode at all times. Note that for numerous applications the active mode describes the operating conditions of a physical process and is driven by external incidents of stochastic nature. The active mode, hence, may not be directly measurable and it may not be available for control purposes at all time instants during the course of operation. When the controller does not have access to any mode information, for achieving stabilization one can resort to adaptive control frameworks (Bercu, Dufour, & Yin, 2009; Caines & Zhang, 1992; Nassiri-Toussi & Caines, 1991) or mode-independent control laws (Boukas, 2006; Vargas, Furloni, & do Val, 2006). On the other hand, if mode information can be observed at certain time instants (even if rarely), this information can be utilized in the control framework. In our earlier work (Cetinkaya & Hayakawa, 2012, 2013b), we investigated stabilization of switched stochastic systems for the case where only *sampled* mode information is available for control purposes. Under the assumption that the active mode is *periodically* observed, we proposed a stabilizing feedback control framework that utilizes the available mode information.

In practical applications, it would be ideal if the mode information of a switched system is available for control purposes at all time instants or at least periodically. However, there are cases where mode information is obtained at *random* time instants. This situation occurs for example when the mode is sampled at all time instants; however, some of the mode samples are randomly lost during communication between mode sampling mechanism and the controller. On the other hand, in some applications, the mode

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has to be detected, but the detected mode information may not always be accurate. In this case each mode detection has a confidence level. Mode information with low confidence is discarded. As a result, depending on the confidence level of detection, the controller may or may not receive the mode information at a particular mode detection instant. In addition, we may also take advantage of random sampling for certain cases and observe the mode intentionally at random instants, as for such cases control under random sampling provides better results compared to periodic sampling. Note that random sampling has also been used for problems such as signal reconstruction and has been shown to have advantages over regular periodic sampling (see Boyle, Haupt, Fudge, & Yeh, 2007, Carlen & Mendes, 2009).

In this paper our goal is to explore the feedback stabilization problem for the case where the active operation mode, which is modeled as a finite-state Markov chain, is observed at *random* time instants. We provide an extended discussion based on our preliminary report (Cetinkaya & Hayakawa, 2013a). Specifically, we assume that the length of intervals between consecutive mode observation instants are identically distributed independent random variables. We employ a renewal process to characterize the occurrences of random mode observations. This characterization allows us to also explore periodic mode observations (Cetinkaya & Hayakawa, 2012, 2013b) as a special case.

We propose a linear feedback control law with a piecewise-constant gain matrix that is switched depending on the value of a randomly sampled version of the mode signal. In order to investigate the evolution of the active mode together with its randomly sampled version, we construct a stochastic process that represents sequences of values the mode takes between random mode observation instants. This sequence-valued stochastic process turns out to be a countable-state Markov chain defined over a set that is composed of all possible mode sequences of finite length. We first analyze the probabilistic dynamics of this sequence-valued Markov chain. Then based on our analysis, we obtain sufficient stabilization conditions for the closed-loop switched stochastic system under our proposed control framework. These stabilization conditions let us assess whether the closed-loop system is stable for a given probability distribution for the length of intervals between consecutive mode observation instants. As this probability distribution is not assumed to have a certain structure, the result presented in this paper can also be considered as a generalization of the result provided in Cetinkaya and Hayakawa (2011), where stabilization problem is discussed in continuous time and the random intervals between mode sampling instants are specifically assumed to be exponentially distributed. In this paper we also explore the case where perfect information regarding the probability distribution for the length of intervals between consecutive mode observation instants is not available. For this problem setting, we present alternative sufficient stabilization conditions which can be used for verifying stability even if the distribution is not exactly known.

The paper is organized as follows. We provide the notation and a review of key results concerning renewal processes in Section 2. In Section 3, we propose our feedback control framework for stabilizing discrete-time switched stochastic systems under randomly available mode information. Then in Section 4, we present sufficient conditions under which our proposed control law guarantees almost sure asymptotic stabilization. In Section 5, we demonstrate the efficacy of our results with two illustrative numerical examples. Finally, in Section 6 we conclude our paper.

2. Mathematical preliminaries

In this section, we provide notation and several definitions concerning discrete-time stochastic processes. Specifically, we denote positive and nonnegative integers by $\mathbb{N}$ and $\mathbb{N}_0$, respectively. Moreover, $\mathbb{R}$ denotes the set of real numbers, $\mathbb{R}^n$ denotes the set of $n \times 1$ real column vectors, and $\mathbb{R}^{n\times m}$ denotes the set of $n \times m$ real matrices. We write $(\cdot)^{\mathrm{T}}$ for transpose. We represent a finite-length sequence of ordered elements $q_1, q_2, \ldots, q_n$ by $q = (q_1, q_2, \ldots, q_n)$. The length (number of elements) of the sequence q is denoted by $|q|$. The notations $\mathbb{P}[\cdot]$ and $\mathbb{E}[\cdot]$ respectively denote the probability and expectation on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with filtration $\{\mathcal{F}_k\}_{k\in\mathbb{N}_0}$. Furthermore, we write $\mathbb{1}_{[G]} : \Omega \to \{0, 1\}$ for the indicator of the set $G \in \mathcal{F}$, that is, $\mathbb{1}_{[G]}(\omega) = 1,\ \omega \in G$, and $\mathbb{1}_{[G]}(\omega) = 0,\ \omega \notin G$.

2.1. Discrete-time renewal processes

A discrete-time renewal process $\{N(k) \in \mathbb{N}_0\}_{k\in\mathbb{N}_0}$ with initial value $N(0) = 0$ is an $\mathcal{F}_k$-adapted stochastic counting process defined by $N(k) \triangleq \sum_{i\in\mathbb{N}} \mathbb{1}_{[t_i\le k]}$, where $t_i \in \mathbb{N}_0,\ i \in \mathbb{N}_0$, are random time instants such that $t_0 = 0$ and $\tau_i \triangleq t_i - t_{i-1} \in \mathbb{N},\ i \in \mathbb{N}$, are identically distributed independent random variables with finite expectation (i.e., $\mathbb{E}[\tau_i] < \infty,\ i \in \mathbb{N}$). Note that $\tau_i,\ i \in \mathbb{N}$, denote the lengths of intervals between time instants $t_i,\ i \in \mathbb{N}_0$. Furthermore, we use $\mu : \mathbb{N} \to [0, 1]$ to denote the common distribution of the random variables $\tau_i,\ i \in \mathbb{N}$, such that

$$\mathbb{P}[\tau_i = \tau] = \mu_\tau, \quad \tau \in \mathbb{N},\ i \in \mathbb{N}, \tag{1}$$

where $\mu_\tau \in [0, 1]$. Note that $\sum_{\tau\in\mathbb{N}} \mu_\tau = 1$. Now, let $\hat{\tau} \triangleq \sum_{\tau\in\mathbb{N}} \tau \mu_\tau = \mathbb{E}[\tau_1](=\mathbb{E}[\tau_i],\ i \in \mathbb{N})$. It follows as a consequence of strong law of large numbers for renewal processes (see Serfozo, 2009) that $\lim_{k\to\infty} \frac{N(k)}{k} = \frac{1}{\hat{\tau}}$.

Note that in Section 3, we employ a renewal process to characterize the occurrences of random mode observations.

2.2. Almost sure asymptotic stability

The zero solution $x(k) \equiv 0$ of a stochastic system is *almost surely stable* if, for all $\epsilon > 0$ and $\rho > 0$, there exists $\delta = \delta(\epsilon, \rho) > 0$ such that if $\|x(0)\| < \delta$, then

$$\mathbb{P}[\sup_{k\in\mathbb{N}_0} \|x(k)\| > \epsilon] < \rho. \tag{2}$$

Furthermore, the zero solution $x(k) \equiv 0$ of a stochastic system is *asymptotically stable almost surely* if it is almost surely stable and

$$\mathbb{P}[\lim_{k\to\infty} \|x(k)\| = 0] = 1. \tag{3}$$

In Sections 3 and 4, we investigate almost sure asymptotic stabilization of a switched stochastic system.

3. Stabilizing switched stochastic systems with randomly available mode information

In this section, we propose a feedback control framework for stabilizing a switched stochastic system by using only the randomly available mode information. Specifically, we consider the discrete-time switched linear stochastic system with $M \in \mathbb{N}$ number of modes given by

$$x(k + 1) = A_{r(k)}x(k) + B_{r(k)}u(k), \quad k \in \mathbb{N}_0, \tag{4}$$

with the initial conditions $x(0) = x_0,\ r(0) = r_0 \in \mathcal{M} \triangleq \{1, 2, \ldots, M\}$, where $x(k) \in \mathbb{R}^n$ and $u(k) \in \mathbb{R}^m$ respectively denote the state vector and the control input; furthermore, $A_i \in \mathbb{R}^{n\times n},\ B_i \in \mathbb{R}^{n\times m},\ i \in \mathcal{M}$, are the subsystem matrices. The mode signal $\{r(k) \in \mathcal{M}\}_{k\in\mathbb{N}_0}$ is assumed to be an $\mathcal{F}_k$-adapted, M-state discrete-time Markov chain with the initial distribution denoted by $\nu : \mathcal{M} \to [0, 1]$ such that $\nu_{r_0} = 1$ and $\nu_i = 0,\ i \neq r_0$.