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A fast time-frequency multi-window analysis using a tuning directional kernel

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In this paper, a novel approach for time-frequency analysis and detection, based on the chirplet transform and dedicated to non-stationary as well as multi-component signals, is presented. Its main purpose is the estimation of spectral energy, instantaneous frequency (IF), spectral delay (SD), and chirp rate (CR) with a high time-frequency resolution (separation ability) achieved by adaptive fitting of the transform kernel. We propose two efficient implementations of this idea, which allow to use the fast Fourier transform (FFT). In the first one, referred to as "self-tuning", a previously proposed CR estimation is used for a local fitting of the chirplet kernel over time. For this purpose, we use the CR associated with the dominant (prominent) component. In the second one, we define a new measure for evaluating at each time-frequency point, how the used analyzing window is matched to the signal. This measure is defined as the absolute difference between the estimated CR and the CR parameter associated to the used analysis window. Our method is able to produce combined time-frequency distributions of the spectral energy, IF, SD, and CR. They are obtained using several classical chirplet transforms with analysis windows of various CRs. The compositions are made by finding the lowest fitting measure for every time-frequency points over all transforms. Finally, we assess the robustness of the methods by a detection application and time-frequency localization, both in the presence of high additive white Gaussian noise (additive white Gaussian noise (AWGN)) as well as we present many time-frequency (TF) images of synthetic and real-world signals.

1. Introduction

Our world is full of non-stationary and multicomponent signals. We say "everything flows" (gr. *panta rei*) after Heraclitus of Ephesus. In a signal processing context, this statement also concerns parameters which describe the signals that surround us. This is the reasons why the development of new methods dedicated to analyze non-stationary and multi-component signals, has a substantial practical importance. Simultaneously, it is commonly known, that time-frequency analysis is one of the most important and powerful approach designed to study real-world signals [1]. In fact, these signals are often non-stationary and consist of numerous components, including noise [2]. Despite many existing approaches [3–10], this paper focuses on well known tools such as the STFT and one of its variants, the chirplet transform (CT) [11,12]. This choice is motivated by results of our recent researches [13,14] which allow to directly use the STFT in order to locally estimate the CR in the TF domain. The resulting distribution can shortly be called "TF phase accelerogram". Our main idea is to use the accelerogram to locally fit the kernel of the CT for each TF point. This adaptive approach can be considered as a contribution to the "matching pursuit" trend [15], since its main purpose is to locally improve the TF resolution (separation ability). Moreover, this approach can be applied iteratively in order to improve the estimation reliability.

Unfortunately, this idea has one fundamental drawback, which is a high computational cost, if the estimated phase accelerogram has different values – which must be assumed for non-stationary signals. Then, the FFT algorithm cannot be applied and the discrete Fourier transform (DFT) has to be directly computed according to its definition for every TF point independently. This can significantly increase the computational cost of the analysis. Therefore, we propose two efficient solutions, which allow a FFT-based implementation. The first one uses a CR-related analysis window adjusted over time. The considered CR should be prominent for a considered analyzed instant or/and can be established by a statistical analysis of a previously computed phase accelerogram with

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its corresponding spectral energy distribution. Both are estimated using STFT whose analyzing window has CR equal to zero. This solution should be suitable for signals with a single dominant component or such, whose components are characterized by a similar time-varying CR, for example, speech. In a second solution, initially, several (or more) CTs, their energy distributions, and accelerograms using FFT for different window CRs have to be computed. Then, we define a simple indicator of "mismatch", and based on it, we introduce a new TF energy distribution directly computed from the CT. The final composed energy distribution can contain energy from the different transforms, however a single TF point represents only one, for which the measure is the smallest over all consider transforms in this point. These implementations can also be used to compute other signal parameters such as IF, SD, and CR.

To summarize this, we try to join two main trends of signal analysis: estimation of local CR in the TF domain and local fitting of the kernel of a time-frequency analyzer [8,16,17] which is the chirplet transformation [11]. Both topics are not new. Many authors proposed interesting methods of CR estimation. Here, we list only selected papers [18–24]. However, we base on our original study and fast estimators introduced in [13,14], which operates in the time-frequency domain directly on STFT (especially on CT) and have evolved since the reassignment approach [25–27]. We can also find some proposition to use the CR for adjusting the analysis window width [28,29]. However, the CR estimation based on CT is presented here the first time.

Hence, this paper is organized as follows. In Section 2, the chirplet transform is introduced with several local CR estimators. In this section, we also define the reassignment operators of the CT and its reassigned time-frequency representation. Then, the new proposed self-tuning chirplet method and multi-window chirplet-based TF distributions are introduced respectively, in Sections 3 and 4. Conclusions are then presented in Section 5.

2. Chirplet-based analysis

The spectral parameters of an analyzed signal $x(t)$, such as IF and CR, can be estimated at any time-frequency coordinates (t, ω) through the STFT as proposed in [13,14]. Herein, we extend this approach to the CT which can be defined by a convolution product with a function g_α expressed as:

$$\begin{aligned} y_x^g(t,\omega) &= \int_{\mathbb{R}} x(\tau) g_\alpha(t-\tau,\omega)\mathrm{d}\tau \\ &= \int_{\mathbb{R}} x(\tau) h(t-\tau)\mathrm{e}^{\mathrm{j}\alpha\frac{(t-\tau)^2}{2}}\mathrm{e}^{\mathrm{j}\omega(t-\tau)}\mathrm{d}\tau \\ &= M_x^g(t,\omega)\mathrm{e}^{\mathrm{j}\phi_x^g(t,\omega)} \end{aligned} \tag{1}$$

where $M_x^g(t,\omega) = \left|y_x^g(t,\omega)\right|$, $\phi_x^g(t,\omega) = \arg\left(y_x^g(t,\omega)\right)$ stand respectively for the magnitude and the phase of the CT, j being the imaginary unit ($\mathrm{j}^2 = -1$) and e being the Euler's number. The kernel of this transformation corresponds to a linear frequency modulated chirp tapered by a real-valued and differentiable analysis window $h(t)$ as follows:

$$g_\alpha(t,\omega) = \underbrace{h(t)\mathrm{e}^{\mathrm{j}\alpha\frac{t^2}{2}}}_{h_\alpha(t)}\mathrm{e}^{\mathrm{j}\omega t} \tag{2}$$

α being the CR parameter of the analysis window. In this paper, we use the 4-term Blackman–Harris window [30] as $h(t)$, because its side lobes are strongly suppressed and even its high order derivatives are easy to compute.

2.1. Chirp rate estimation

As proposed in [13], a TF phase accelerogram can be estimated through the amplitude of the STFT. Following this idea and using notations consistent with [14], we can propose a new CR estimator based on the CT as defined by Eq. (1) using specific analysis windows $\mathcal{D}g_\alpha(t,\omega)$ and $\mathcal{T}g_\alpha(t,\omega)$ as:

$$\hat{R}_x^g(t,\omega) = \frac{\Re\left(\frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\right)}{\Im\left(\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right)}, \tag{3}$$

where $\Re()$ and $\Im()$ return respectively, the real and the imaginary parts of a complex number. In the corresponding software [31], we simply denote this estimator as "K". $\mathcal{D}g_\alpha(t,\omega)$ is the first-order derivative of the window $g_\alpha(t, \omega)$ with respect to time:

$$\mathcal{D}g_\alpha(t,\omega) = \frac{\partial g_\alpha(t,\omega)}{\partial t} = \left(\frac{dh_\alpha}{dt}(t) + \mathrm{j}\omega h_\alpha(t)\right)\mathrm{e}^{j\omega t} \tag{4}$$

as well as $\mathcal{T}g_\alpha(t,\omega)$ is the product of the window $g_\alpha(t, \omega)$ and a time ramp function which is simply the linear odd function with a slope equal to one

$$\mathcal{T}g_\alpha(t,\omega) = tg_\alpha(t,\omega). \tag{5}$$

Since we proposed in [14] enhanced CR estimators based on higher-order derivatives, we can also deduce similar ones by applying L'Hôpital's rule on Eq. (3). These new estimators can be expressed as follows:

$$\hat{\hat{R}}_x^g(t,\omega) = \frac{\frac{\partial}{\partial t}\left[\Re\left(\frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\right)\right]}{\frac{\partial}{\partial t}\left[\Im\left(\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right)\right]} = \frac{\Re\left(\frac{y_x^{\mathcal{D}^2g}(t,\omega)}{y_x^g(t,\omega)} - \frac{y_x^{\mathcal{D}g}(t,\omega)^2}{y_x^g(t,\omega)^2}\right)}{\Im\left(\frac{y_x^{\mathcal{D}\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)} - \frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right)} \tag{6}$$

as well as

$$\hat{\hat{R}}_x^g(t,\omega) = \frac{\frac{\partial}{\partial \omega}\left[\Re\left(\frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\right)\right]}{\frac{\partial}{\partial \omega}\left[\Im\left(\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right)\right]} = -\frac{\Im\left(\frac{y_x^{\mathcal{D}\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)} - \frac{y_x^{\mathcal{D}g}(t,\omega)}{y_x^g(t,\omega)}\frac{y_x^{\mathcal{T}g}(t,\omega)}{y_x^g(t,\omega)}\right)}{\Re\left(\frac{y_x^{\mathcal{T}^2g}(t,\omega)}{y_x^g(t,\omega)} - \frac{y_x^{\mathcal{T}g}(t,\omega)^2}{y_x^g(t,\omega)^2}\right)}, \tag{7}$$

where $\mathcal{D}^2g_\alpha(t,\omega) = \frac{\partial^2 g_\alpha(t,\omega)}{\partial t^2}$, $\mathcal{T}^2g_\alpha(t,\omega) = t^2g_\alpha(t,\omega)$ and $\mathcal{D}\mathcal{T}g_\alpha(t,\omega) = g_\alpha(t,\omega) + t\mathcal{D}g_\alpha(t,\omega)$. In the corresponding software [31], we simply denote this estimators, respectively, as "D" and "F". These estimators obtain better results when they are applied close to the component attractors identified in the TF domain by their instantaneous frequencies and spectral delays [2,14,25,26,32,33]. However, despite the spectral energy is often concentrated in these areas, both the numerator and denominator of Eq. (3) can go to zero causing numerical instabilities. Therefore, estimators (6) and (7) were shown to be more robust than (3) [14].

2.2. Directional kernel

The concept of the directional kernel (TF-oriented) in the context of adaptive TF analysis is still developed [8]. In very few words, this approach is closely associated with the ambiguity function of the analysis window used by CT which is defined by

$$A(\tau,\nu) = \int_{\mathbb{R}} h_\alpha(t)h_\alpha(t-\tau)^*\mathrm{e}^{j\nu t}\,dt. \tag{8}$$

Significant values of this function are distributed into the ambiguity area according to the window envelope $h(t)$ and its CR α as illustrated in Fig. 1. Especially, the CR indicates a direction in the TF domain. The energy of each component, whose attractor is distributed along to this direction, is well concentrated in the spectrogram obtained using this window. If the CR is exactly equal to this direction, the window is locally matched to this component. This aspect highlights the importance of a proper selection of the window and of its parameters, especially its CR α.