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Brief paper

Adaptive output feedback control for a class of large-scale nonlinear time-delay systems[☆]

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ABSTRACT

This paper addresses the problem of global adaptive stabilization by output feedback for a class of large-scale nonlinear time-delay systems. With a new Lyapunov–Krasovskii functional, a memoryless adaptive output feedback controller is developed. The precise knowledge of the time-varying delay is not required to be known *a priori*. It is proved that global stability of the large-scale nonlinear time-delay system can be achieved by the proposed approach. A numerical example is given to illustrate the effectiveness of the proposed approach.

1. Introduction

Control of large-scale systems has received considerable attention during the past decade; see, for instance, Chen and Li (2008), Liu, Jiang, and Hill (2012); Liu, Zhang, and Jiang (2007), Mehraeen, Jagannathan, and Crow (2011) and the references cited therein, in which backstepping technique (Krstic, Kanellakopoulos, & Kokotovic, 1995) was commonly used and well applied to robust and adaptive control of large-scale systems. Based on backstepping design, the problem of adaptive stabilization by output feedback was investigated in Liu et al. (2007) for a class of large-scale systems. In Liu et al. (2012), the authors considered output feedback control of large-scale nonlinear systems in the presence of non-smooth sensor noise.

It is well known that time-delay phenomenon widely exists in networked control systems, communication networks, chemical reactors, rolling mills, etc. Therefore, much recent research has been devoted to control of time-delay systems (Ge, Hong, & Lee, 2005; Hua, Guan, & Shi, 2005; Ibrir, 2011; Jiao & Shen, 2005; Zhou, Shi, Xu, & Li, 2013) and then large-scale time-delay systems as well (Tong, Li, & Zhang, 2011; Ye, 2011; Yoo & Park, 2012; Zhang, Liu, Feng, & Zhang, 2013; Zhou, 2008). In Zhou (2008), an adaptive control scheme was proposed to address output tracking of a class of interconnected time-delay subsystems. An adaptive output-feedback stabilizer was proposed in Ye (2011) for a class of large-scale nonlinear time-delay systems without a priori knowledge of subsystems high-frequency-gain signs. The results of Tong et al. (2011), Ye (2011) and Zhou (2008), however, can only be applied to systems whose bounding functions of interaction time-delay terms are only output nonlinearities. With the employment of neural networks, the authors of Yoo and Park (2012) focused on the adaptive output feedback controller design for large-scale nonlinear systems with the bounding functions of interaction time-delay terms including all states of subsystems, but only semiglobal stability can be guaranteed. Recently, based on dynamic gain scaling technique (Krishnamurthy & Khorrami, 2004; Praly & Jiang, 2004), stabilization problem was solved by Zhang et al. (2013) for a class of large-scale nonlinear time-delay systems in lower triangular form, in which the uncertain nonlinearities were assumed to be bounded by continuous functions of the outputs or delayed outputs multiplied by unmeasured states or delayed states. However, the output feedback controller is a delay-dependent one, i.e., part of the time-delay nonlinearities is dealt with by the controller directly, and the precise knowledge of the time-varying delay needs to be known. Meanwhile, to design a memoryless output feedback controller, Zhang et al. (2013) introduced a much more restrictive condition requiring some of the system nonlinearities to satisfy linear growth condition while the Lyapunov–Krasovskii functional and the design procedure were not changed essentially.

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In this paper, we consider global adaptive stabilization by output feedback for a class of large-scale nonlinear time-delay systems. Our main contributions are summarized below.

- By introducing quadratic-like terms, a new Lyapunov–Krasovskii functional is proposed, with which all the time-delay nonlinearities can be eliminated such that a memoryless output feedback controller can be successfully constructed.
- The considered large-scale time-delay system satisfies a very general growth condition and the precise knowledge of the time-delay is not required to be known *a priori*. Note that in Zhang et al. (2013), a more restrictive condition is necessary to design the memoryless controller.
- Parameter uncertainty is taken into account. Therefore, some nonlinear time-delay systems considered in those adaptive control schemes (Tong et al., 2011; Zhou, 2008) can be included.

The rest of the paper is organized as follows. In Section 2, we state the control objective and two technical lemmas. In Section 3, the adaptive output feedback controller design is presented. Stability analysis is given in Section 4. Simulation results are shown in Section 5.

2. Problem statement and preliminaries

In this paper, we consider a class of large-scale nonlinear time-delay systems composed of N subsystems[1]:

$$\dot{x}_{i,j} = x_{i,j+1} + f_{i,j}\left(t, x, x\left(t - d\left(t\right)\right), \theta_{i,j}\right),$$
$$y_i = x_{i,1}, \quad j = 1, \ldots, n_i, \ i = 1, \ldots, N, \tag{1}$$

where $x_i\left(t\right) = \left[x_{i,1}\left(t\right), \ldots, x_{i,n_i}\left(t\right)\right]^T \in R^{n_i}$ $(n_i \geq 2)$ are the state vectors, $x_{i,n_i+1} = u_i \in R$ and $y_i \in R$ are the input and output of the ith subsystem, respectively, the nonnegative bounded function $d\left(t\right)$ denotes the unknown time-varying delay satisfying $\dot{d}\left(t\right) \leq \bar{d} < 1$ for a known constant $\bar{d}$, $\theta_{i,j} \in R^m$ are unknown parameters belonging to an unknown compact set Ω, $x\left(t\right) = \left[x_1^T\left(t\right), \ldots, x_N^T\left(t\right)\right]^T$, $y = \left[y_1, \ldots, y_N\right]^T$, and the uncertain continuous functions $f_{i,j} : R^+ \times R^{n_1+\cdots+n_N} \times R^{n_1+\cdots+n_N} \times \Omega \to R$ are locally Lipschitz in $(x, x\left(t - d\left(t\right)\right))$ uniformly in t and $\theta_{i,j}$ which guarantees existence and uniqueness of the solutions.

For system (1), we make the following assumption.

A.1: For all $\left(t, x, x\left(t - d\left(t\right)\right), \theta_{i,j}\right) \in R^+ \times R^{n_1+\cdots+n_N} \times R^{n_1+\cdots+n_N} \times \Omega$, the following inequalities hold:

$$\left|f_{i,1}\left(t, x, x\left(t - d\left(t\right)\right), \theta_{i,j}\right)\right| \leq \sum_{p=1}^{N}\Big(\theta_0\gamma_{i,1}\left(y_p\right)\left|y_p\right| + \theta_0\tilde{\gamma}_{i,1}\left(y_p\left(t - d\left(t\right)\right)\right)\left|y_p\left(t - d\left(t\right)\right)\right|\Big),$$

$$\left|f_{i,j}\left(t, x, x\left(t - d\left(t\right)\right), \theta_{i,j}\right)\right| \leq \sum_{p=1}^{N}\Big(\theta_0\gamma_{i,j}\left(y\right)\left|y_p\right| + \theta_0\tilde{\gamma}_{i,j}\left(y, y\left(t - d\left(t\right)\right)\right)\left|y_p\left(t - d\left(t\right)\right)\right|\Big) + \sum_{p=1}^{N}\sum_{q=2}^{\min\{n_p,j\}}\Big(\gamma_{i,j}\left(y\right)\left|x_{p,q}\right| + \tilde{\gamma}_{i,j}\left(y, y\left(t - d\left(t\right)\right)\right)\left|x_{p,q}\left(t - d\left(t\right)\right)\right|\Big),$$
$$j = 2, \ldots, n_i, \ i = 1, \ldots, N, \tag{2}$$

where θ_0 is an unknown nonnegative constant, and $\gamma_{i,j}\left(\cdot\right)$ and $\tilde{\gamma}_{i,j}\left(\cdot\right)$, $j = 1, \ldots, n_i$, are known continuous functions in their arguments.

Remark 1. Usually, θ_0 can be viewed as the upper bound of $\theta_{i,j}$ for many common system nonlinearities. When θ_0 is known, Assumption A.1 is a very similar growth condition to Assumption 1 of Zhang et al. (2013). The only difference is that $\gamma_{i,j}(y), j = 2, \ldots, n_i$, are not functions of $(y, y(t - d(t)))$. The same as Zhang et al. (2013), the system (1) covers a wide variety of nonlinear systems in the literature. Moreover, in the presence of parameter uncertainty, the system (1) can also include some nonlinear time-delay systems considered in the adaptive control schemes (Tong et al., 2011; Zhou, 2008). Note that Assumption A.1 is a more general condition than Assumption 2 of Zhang et al. (2013). Indeed, compared with the growth conditions for memoryless output feedback controller design in the literature, the condition in Assumption A.1 has been much more relaxed.

The main purpose of this paper is to design a memoryless adaptive output feedback controller for the time-delay system (1) satisfying Assumption A.1 such that the system state x converges to the origin while all the closed loop states are bounded.

First of all, we introduce two technical lemmas that are useful for the controller design and stability analysis.

Lemma 1 (*Lin & Qian, 2002*). *For any real-valued continuous function $a(x, y)$, there are smooth functions $a_1(x) \geq 0$ and $a_2(y) \geq 0$, such that*

$$|a(x, y)| \leq a_1(x)a_2(y),$$
$$|a(x, y)| \leq a_1(x) + a_2(y). \tag{3}$$

Lemma 2 (*Praly & Jiang, 2004*). *For $i = 1, \ldots, N$, there exist real numbers $\sigma_{i,1}$ and $\sigma_{i,2}$, symmetric matrices $P_{i,1}$ and $P_{i,2}$, and column vectors $a_i = [a_{i,1}, \ldots, a_{i,n_i-1}]^T$ and $k_i = [k_{i,1}, \ldots, k_{i,n_i-1}]^T$ satisfying the following set of inequalities*

$$\sigma_{i,1} > 0, \quad \sigma_{i,2} \geq 0, \quad P_{i,1} > 0, \quad P_{i,2} > 0,$$
$$-\sigma_{i,1}I \geq \left(A_i - a_ic_i^T\right)^T P_{i,1} + P_{i,1}\left(A_i - a_ic_i^T\right),$$
$$-2\sigma_{i,1}I \geq \left(A_i - b_ik_i^T\right)^T P_{i,2} + P_{i,2}\left(A_i - b_ik_i^T\right),$$
$$\frac{1}{2}P_{i,1} \leq D_iP_{i,1} + P_{i,1}D_i + P_{i,1} \leq \sigma_{i,2}P_{i,1},$$
$$\frac{1}{2}P_{i,2} \leq D_iP_{i,2} + P_{i,2}D_i + P_{i,2} \leq \sigma_{i,2}P_{i,2}, \tag{4}$$

where $D_i = \mathrm{diag}\{0, 1, \ldots, n_i - 2\}$, and A_i, b_i, c_i are defined as

$$A_i = \begin{bmatrix} 0 & I_{n_i-2} \\ 0 & 0 \end{bmatrix}, \quad b_i = \begin{bmatrix} 0_{(n_i-2)\times 1} \\ 1 \end{bmatrix}, \quad c_i = \begin{bmatrix} 1 \\ 0_{(n_i-2)\times 1} \end{bmatrix}. \tag{5}$$

Remark 2. Lemma 1 is used to deal with the system nonlinearities while Lemma 2, to determine some important design parameters.

3. Adaptive output feedback design

In this section, we design an adaptive output feedback controller for the nonlinear time-delay system. Specifically, the following reduced-order observer and controller are constructed for the ith subsystem of (1):

$$\begin{aligned} \dot{\hat{x}}_{i,j+1} &= \hat{x}_{i,j+2} - l^ja_{i,j}\hat{x}_{i,2} + l^ja_{i,j}\hat{\theta}_i\rho_i\left(y_i\right)y_i \\ &\quad - l^{j+1}a_{i,1}a_{i,j}y_i - jl^{j-1}\dot{l}a_{i,j}y_i + l^{j+1}a_{i,j+1}y_i, \\ &\quad j = 1, \ldots, n_i - 2, \\ \dot{\hat{x}}_{i,n_i} &= u_i - l^{n_i-1}a_{i,n_i-1}\hat{x}_{i,2} + l^{n_i-1}a_{i,n_i-1}\hat{\theta}_i\rho_i\left(y_i\right)y_i \\ &\quad - l^{n_i}a_{i,1}a_{i,n_i-1}y_i - \left(n_i - 1\right)l^{n_i-2}\dot{l}a_{i,n_i-1}y_i, \\ u_i &= -l^{n_i-1}k_{i,1}\left(\hat{x}_{i,2} + la_{i,1}y_i\right) - \cdots \\ &\quad - lk_{i,n_i-1}\left(\hat{x}_{i,n_i} + l^{n_i-1}a_{i,n_i-1}y_i\right), \end{aligned} \tag{6}$$

[1] Throughout the paper, for simplicity, we will drop the argument of some functions whenever no confusion can arise; for instance, $x_{i,j}$ is used to denote $x_{i,j}\left(t\right)$.