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Short communication

# Sign normalised spline adaptive filtering algorithms against impulsive noise

Chang Liu*, Zhi Zhang, Xiao Tang

*School of Electronic Engineering, Dongguan University of Technology, 1 DaXueRoad Dongguan, 523808, China*

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ABSTRACT

In this paper, a sign normalised least mean square algorithm (SNLMS) based on Wiener spline adaptive filter, called SAF-SNLMS, is proposed. The proposed algorithm is derived by minimising the absolute value of the a posteriori error. Moreover, to further improve the convergence performance of the SAF-SNLMS, the variable step-size scheme is introduced. Simulation results demonstrate the SAF-SNLMS and its variable step-size variant obtain more robust performance when compared with the existing spline adaptive filter algorithms in impulsive noise.



## 1. Introduction

The merits of the linear adaptive filter are its simple design and analysis, which lead to its wide application in many practical engineering problems such as acoustic echo cancellation (AEC), acoustic noise control (ANC), channel estimation and equalization. For the linear adaptive filter, its weight coefficients can be updated by using several sophisticated adaptive algorithms like the least mean square (LMS) algorithm, normalized least mean square (NLMS) algorithm and affine projection algorithm (APA). However, the linear model suffers from the performance degradation because of the failure to model the nonlinearity.

In recent years, in order to model the nonlinearity, several adaptive nonlinear spline adaptive filters (SAFs) have been introduced, such as Wiener spline filter [1], Hammerstein spline filter [2] and cascade spline filter [3]. The nonlinearity in this kind of structure is modeled by an adaptive look-up table (LUT) in which the control points are interpolated by a local low order polynomial spline curve. The adaptive spline filters achieve improved performance in modelling the nonlinearity. However, since their adaptation is derived by minimising the squared value of the instantaneous error, the performance of the spline adaptive filter can deteriorate seriously in impulsive noise. To alleviate this problem, sign adaptive algorithm is an excellent candidate. The weight vector in sign adaptive algorithm is commonly updated in accordance with the $L_1$ norm optimization criterion. The affine projection sign algorithm was proposed in [4] which guaranteed the robustness against impulsive noise. In addition, several sign subband adaptive filters with variable step-size were introduced to improve the convergence speed and combat the impulsive noise [5–7].

In this brief paper we extend the sign idea into spline adaptive filter and propose a new sign normalised least mean square algorithm based on Wiener spline adaptive filter which is called SAF-SNLMS. It is derived by minimising the absolute value of the a posteriori error and used to identify the Wiener-type nonlinear systems. Furthermore, by adjusting the step-size associated with the squared value of the impulsive-free error, the variable step-size SAF-SNLMS (SAF-VSS-SNLMS) algorithm is proposed. It is demonstrated that the proposed algorithms offer better convergence performance and robustness compared with the conventional SAF algorithms in the impulsive noise environment.

## 2. SAF-NLMS algorithm

Fig. 1 shows the structure of SAF [1,8], assuming that the input of the SAF at time $(n)$ is $x(n)$, $s(n)$ represents the output of the linear network which is given by

$$s(n) = \mathbf{w}^T(n)\mathbf{x}(n), \tag{1}$$

where $\mathbf{w}(n) = [w(0), w(1), \cdots, w(M-1)]^T$ represents the weight vector of the FIR filter with length $M$, and $\mathbf{x}(n) = [x(n), x(n-1), \cdots, x(n-M+1)]^T$ is the input vector of the linear network.

With reference to the spline interpolation scheme in [1], third-order spline curves are applied, thus the output of the whole

* Corresponding author.
*E-mail address:* chaneaaa@163.com (C. Liu).

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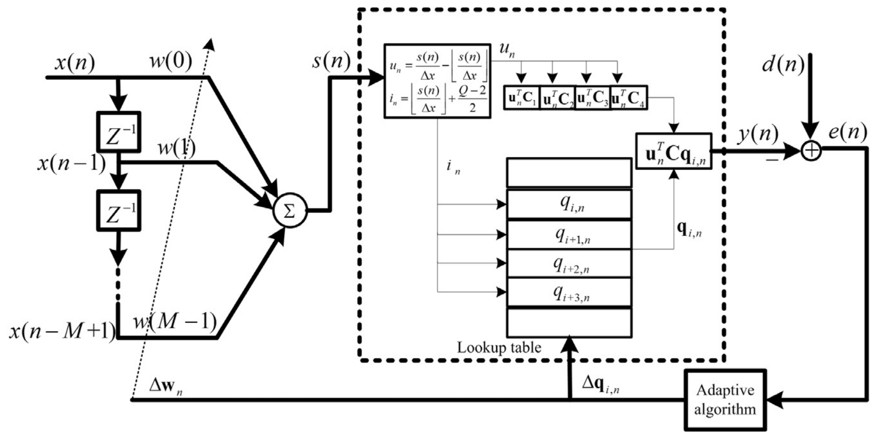


**Fig. 1.** Structure of SAF [1,8].

system $y(n)$ can be expressed as

$$y(n) = \varphi_i(u_n) = \mathbf{u}_n^T \mathbf{C} \mathbf{q}_{i,n}, \tag{2}$$

where $\mathbf{u}_n = [u_n^3, u_n^2, u_n, 1]^T$, $\mathbf{q}_{i,n} = [q_{i,n}, q_{i+1,n}, q_{i+2,n}, q_{i+3,n}]^T$ is the control point vector. The superscript $T$ denotes the transposition operation. $\mathbf{C}$ is the spline basis matrix whose dimension is $4 \times 4$. Two suitable types of spline basis are Catmul-Rom (CR) spline and B-spline whose spline basis matrices are given by

$$C_B = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}, C_{CR} = \frac{1}{2}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 2 & -5 & 4 & -1 \\ -1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 0 \end{bmatrix}, \tag{3}$$

and the span index $i$ and the local parameter $u_n$ can be defined as follows

$$u_n = \frac{s(n)}{\Delta x} - \left\lfloor \frac{s(n)}{\Delta x} \right\rfloor, \tag{4}$$

$$i = \left\lfloor \frac{s(n)}{\Delta x} \right\rfloor + \frac{Q-1}{2}, \tag{5}$$

where $\Delta x$ is the uniform space between two adjacent control points, $Q$ is the total number of control point and $\lfloor \cdot \rfloor$ denotes the floor operator.

Using the Lagrange multiplier method, the cost function for the SAF-NLMS can be defined as [8]

$$\Theta_0(\mathbf{q}_{i,n+1}) = \frac{1}{2\mathbf{u}_n^T \mathbf{u}_n} e^2(n) + \frac{1}{2}||\mathbf{q}_{i,n+1} - \mathbf{q}_{i,n}||^2, \tag{6}$$

where $(1/2) \times [e(n)/\mathbf{u}_n^T \mathbf{u}_n]$ can be viewed as the Lagrange multiplier [8]. $e(n)$ is the a priori error which can be expressed as $e(n) = d(n) - y(n) = d(n) - \mathbf{u}_n^T \mathbf{C} \mathbf{q}_{i,n}$, and $d(n)$ is the desired signal which contains impulsive noise.

Taking the derivative of (6) with respect to $\mathbf{q}_{i,n+1}$ and $\mathbf{w}_{n+1}$ respectively, and setting them to zeros, we can obtain two recursive equations of the tap weights and control points for the NLMS-SAF algorithm [8]

$$\mathbf{w}_{n+1} = \mathbf{w}_n + \mu_{\mathrm{w}} \frac{e(n)}{\mathbf{u}_n^T \mathbf{u}_n + \varepsilon} \frac{1}{\Delta x} \dot{\mathbf{u}}_n^T \mathbf{C} \mathbf{q}_{i,n} \mathbf{x}_n, \tag{7}$$

$$\mathbf{q}_{i,n+1} = \mathbf{q}_{i,n} + \mu_{\mathrm{q}} \frac{e(n)}{\mathbf{u}_n^T \mathbf{u}_n + \varepsilon} \mathbf{C}^T \mathbf{u}_n, \tag{8}$$

where $\mu_{\mathrm{w}}$ and $\mu_{\mathrm{q}}$ are the step-sizes for the linear network and nonlinear network respectively, the small positive constant $\varepsilon$ is used for avoiding zero-division.

## 3. Proposed sign SAF-NLMS algorithms

### 3.1. SAF-SNLMS algorithm

The updating equation of $\mathbf{q}_{i,n}$ in the proposed sign SAF-NLMS algorithm can be formulated by the following constrained optimization problem:

$$\min_{\mathbf{q}_{i,n+1}} |e_p(n)| = |d(n) - y(n+1)| = |d(n) - \mathbf{u}_n^T \mathbf{C} \mathbf{q}_{i,n+1}| \quad \text{subject to } ||\mathbf{q}_{i,n+1} - \mathbf{q}_{i,n}||^2 \le \beta^2, \tag{9}$$

where $e_p(n) = d(n) - \mathbf{u}_n^T \mathbf{C} \mathbf{q}_{i,n+1}$ is defined as the a posteriori error, $\beta^2$ is selected to be a small parameter ensuring the updating of $\mathbf{q}_{i,n}$ dose not change drastically, $|\cdot|$ is the absolute value operation and $||\cdot||$ denotes the Eucilidean norm of a vector.

Then, using the Lagrange multiplier method, the cost function can be expressed by

$$\Theta(\mathbf{q}_{i,n+1}) = |e_p(n)| + \rho_0[||\mathbf{q}_{i,n+1} - \mathbf{q}_{i,n}||^2 - \beta^2], \tag{10}$$

where $\rho_0$ denotes the Lagrange multiplier. Setting the derivative of the cost function $\Theta(\mathbf{q}_{i,n+1})$ with respect to $\mathbf{q}_{i,n+1}$ equal to zero, we have

$$\mathbf{q}_{i,n+1} = \mathbf{q}_{i,n} + \frac{1}{2\rho_0} \mathbf{C}^T \mathbf{u}_n \mathrm{sgn}[e_p(n)], \tag{11}$$

where sgn[.] is the sign function.

Substituting (11) into the constraint condition in (9), we obtain

$$\frac{1}{2\rho_0} = \frac{\beta}{||\mathbf{C}^T \mathbf{u}_n||}, \tag{12}$$

Note that $\mathbf{C}^T$ is a constant matrix and $||\mathbf{C}^T \mathbf{u}_n|| \le ||\mathbf{C}^T|| \cdot ||\mathbf{u}_n||$, where $||\mathbf{C}^T||$ is defined as the spectral norm of matrix $\mathbf{C}^T$, $||\mathbf{C}^T|| := \sup_{\mathbf{u}_n \neq 0}[||\mathbf{C}^T \mathbf{u}_n||/||\mathbf{u}_n||]$. Thus, (12) can be rewritten as

$$\frac{1}{2\rho_0} \ge \frac{\beta_0}{\sqrt{\mathbf{u}_n^T \mathbf{u}_n + \varepsilon_0}}, \tag{13}$$

where $\beta_0 = \beta/||\mathbf{C}^T||$ and $\varepsilon_0$ is small positive constant used for avoiding zero-division.

Considering the lower bound of $1/(2\rho_0)$ in (13), the updating equation of $\mathbf{q}_{i,n}$ can be derived as

$$\mathbf{q}_{i,n+1} = \mathbf{q}_{i,n} + \mu_{\mathrm{q}} \frac{\mathrm{sgn}[e_p(n)]}{\sqrt{\mathbf{u}_n^T \mathbf{u}_n + \varepsilon_0}} \mathbf{C}^T \mathbf{u}_n \tag{14}$$

In a similar manner, the cost function associated with the weight vector of FIR filter $\mathbf{w}_n$ can be formulated as

$$J(\mathbf{w}_{n+1}) = |e_p(n)| + \rho_0[||\mathbf{w}_{n+1} - \mathbf{w}_n||^2 - \beta^2], \tag{15}$$