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Brief paper

Optimal robust filtering for systems subject to uncertainties☆

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ABSTRACT

In this paper we deal with an optimal filtering problem for uncertain discrete-time systems. Parametric uncertainties of the underlying model are assumed to be norm bounded. We propose an approach based on regularization and penalty function to solve this problem. The optimal robust filter with the respective recursive Riccati equation is written through unified frameworks defined in terms of matrix blocks. These frameworks do not depend on any auxiliary parameters to be tuned. Simulation results show the effectiveness of the robust filter proposed.

1. Introduction

The Kalman filter (Anderson & Moore, 1979; Kailath, Sayed, & Hassibi, 2000; Kalman, 1960) has been widely used to solve estimation problems in attitude and position determination, robotics, communications, control, economics, signal processing, computer vision and other fields, see for instance Brown and Hwang (1997), Farrel (2008), Hasan and Azim-Sadjani (1995), Mills and Goldenberg (1989) and Stevens and Lewis (1991). It was developed in the 1960s based on the assumption that all parameter matrices of the state-space model are not subject to uncertainties. This assumption guarantees optimal state estimates, essential to describe the essence of dynamic systems. However, when the model considered in the filtering process is uncertain this central premise of the Kalman filter is violated. In this case, its performance can be severely degraded. One of the first studies on the sensibility of the Kalman filter in the presence of parameter uncertainties was performed in D'Appolito and Hutchinson (1969).

In the last decades this problem has motivated the development of robust estimation approaches to limit the performance degradation of optimal filters, see for instance Einicke and White (1999), Sayed (2001), Shaked and de Souza (1995), Wang and Balakrishnan (2002), Zhou (2010) and references therein. Four representative approaches that deal with these recursive estimation problems were proposed based on $\mathcal{H}_\infty$ filtering (Hassibi, Sayed, & Kailath, 1999), set-valued estimation (Bertsekas & Rhodes, 1971), guaranteed-cost (Petersen & Savkin, 1999; Xie, Soh, & de Souza, 1994), and robust regularized least-squares (Sayed, 2001). All these approaches were compared in Sayed (2001), where relevant questions on parameterization, stability, robustness, and online applications were considered.

The first three filters presented in Bertsekas and Rhodes (1971), Hassibi et al. (1999), Petersen and Savkin (1999) and Xie et al. (1994) were not deduced based on regularization techniques. The existence conditions of them should be checked at every instant of time when we are dealing with time-varying systems. Auxiliary parameters should be tuned in order to guarantee stability and optimal performance.

The filters introduced in Sayed (2001), and also the filters given in Ishihara and Terra (2008), Ishihara, Terra, and Campos (2005, 2006) and Terra, Ishihara, and Padoan (2007), were deduced based on regularization approaches. They aim to minimize the worst-possible residual norm over an admissible class of uncertainties at each iteration. Similar to the standard Kalman filter, they are useful to be used in online applications. In these cases, the stability can be always guaranteed. However, to obtain an optimal robust performance a Lagrange multiplier should be tuned in order to minimize a scalar unimodal function.

In this paper, we propose a robust recursive filter for linear systems subject to norm-bounded parameter uncertainties based on robust regularized least-squares problem (Sayed & Nascimento,

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1999) and penalty function (Luenberger, 2003). The combination of both techniques aims to include in a convex cost function all data related to the state and measurement equations which model the uncertainties of the system. A penalty function parameter imposes that the argument of the robust regularization vanishes. As a consequence of this penalization, the minimization of the cost function is obtained without the need of performing offline computations.

The robust filter developed in this paper does not depend on any auxiliary parameter to be tuned, only on the parameters and weighting matrices known *a-priori*. The recursiveness provides a computational advantage, in terms of online implementation, in virtue of their similarities with the standard Kalman filter. The stability and convergence are guaranteed for steady-state estimates.

We present a numerical example to show the evaluation of the poles of the optimal robust filter proposed in order to demonstrate the influence of the penalty parameter in the convergence and stability of the filter (see Eq. (1) in Box I).

The rest of this paper is organized as follows. The filtering problem dealt with in this paper is defined in Section 2, preliminary results are shown in Section 3, optimal robust estimates are developed in Section 4, stability and convergence of the robust filter are presented in Section 5, numerical examples are given in Section 6 and some concluding remarks are provided in Section 7.

The notation we use in this paper is standard. $\mathbb{R}$ is the set of real numbers, $\mathbb{R}^n$ is the set of n-dimensional vectors whose elements are in $\mathbb{R}$, $\mathbb{R}^{m\times n}$ is the set of $m \times n$ real matrices, A^T is the transpose of the matrix A, $P \succ 0$ ($P \succeq 0$) denotes a positive definite (semidefinite) matrix, $\|x\|$ is the Euclidean norm of x, $\|x\|_P$ is the weighted norm of x defined by $(x^TPx)^{\frac{1}{2}}$, the notation $Y^TXY = Y^TX(\cdot)$ is adopted for convenience and I_n represents $n \times n$ identity matrices.

2. Robust estimation problem

Consider the uncertain discrete-time dynamic system:

$$x_{k+1} = (F_k + \delta F_k)\, x_k + (G_k + \delta G_k)\, w_k, \qquad (2)$$
$$z_{k+1} = (H_{k+1} + \delta H_{k+1})\, x_{k+1} + (K_{k+1} + \delta K_{k+1})\, v_{k+1},$$

for all $k \geq 0$, where $x_k \in \mathbb{R}^n$ is the state vector, $z_{k+1} \in \mathbb{R}^p$ is the output vector, w_k is the state noise vector, and v_{k+1} is the output noise vector, $F_k \in \mathbb{R}^{n\times n}$, $G_k \in \mathbb{R}^{n\times m}$, $H_{k+1} \in \mathbb{R}^{p\times n}$, $K_{k+1} \in \mathbb{R}^{p\times m}$ are nominal parameter matrices, $\delta F_k \in \mathbb{R}^{n\times n}$, $\delta G_k \in \mathbb{R}^{n\times m}$, $\delta H_{k+1} \in \mathbb{R}^{p\times n}$, $\delta K_{k+1} \in \mathbb{R}^{p\times m}$ are uncertain matrices with

$$\begin{bmatrix}\delta F_k & \delta G_k\end{bmatrix} = M_{1,k}\Delta_1 \begin{bmatrix}N_{F_k} & N_{G_k}\end{bmatrix}, \qquad \|\Delta_1\| \leq 1,$$
$$\begin{bmatrix}\delta H_{k+1} & \delta K_{k+1}\end{bmatrix} = M_{2,k}\Delta_2 \begin{bmatrix}N_{H_{k+1}} & N_{K_{k+1}}\end{bmatrix}, \qquad \|\Delta_2\| \leq 1,$$

Δ_1 and Δ_2 are arbitrary contractions. As usual, x_0, $\{w_k\}$, and $\{v_{k+1}\}$ are assumed mutually independent zero-mean Gaussian random variables with variances $\mathbb{E}\{x_0x_0^T\} = \Pi_0 \succ 0$, $\mathbb{E}\{w_kw_k^T\} = Q_k \succ 0$, and $\mathbb{E}\{v_{k+1}v_{k+1}^T\} = R_{k+1} \succ 0$, respectively. A recursive optimal robust filter is proposed based on the solution $\hat{x}_{k+1|k+1}(\mu)$ of the optimization problem:

$$\min_{x_f} \max_{\delta_f} \left\{ \mathcal{J}_k^{\mu}(x_f, \delta_f) \right\}, \qquad (3)$$

with the cost function $\mathcal{J}_k^{\mu}(x_f, \delta_f)$ defined in (1), where $x_f := (x_k, w_k, v_{k+1}, x_{k+1})$ and $\delta_f := \{\delta F_k, \delta G_k, \delta K_{k+1}, \delta H_{k+1}\}$, $F_{\delta,k} = F_k + \delta F_k$, $G_{\delta,k} = G_k + \delta G_k$, $H_{\delta,k+1} = H_{k+1} + \delta H_{k+1}$, $K_{\delta,k+1} = K_{k+1} + \delta K_{k+1}$, and $\mu > 0$. The matrices $P_{k|k} \succ 0$, Q_k, and R_{k+1} are weighting matrices and μ is a penalty parameter. We assume that we have an *a-priori* estimation for each step k for the state x_k denoted by $\hat{x}_{k|k}$ along with the observation at time $(k+1)$ given by z_{k+1}.

The formulation (3) is motivated by the fact that stochastic estimation problems can also be solved through deterministic arguments (e.g., Bryson & Ho, 1975, Cox, 1964, Larson & Peschon, 1966 and Sayed, 2001). Since the filtering problem dealt with in this paper consists in obtaining the best state estimate in contrast to the worst influence of uncertainties, the penalty parameter is able to encompass the whole uncertainties of the system. However, if uncertainties are not considered, (3)–(1) reduces to the following minimization problem:

$$\min_{x_f} \left\{ J_k^{\mu}\left(x_f\right) \right\} \qquad (4)$$

with

$$\begin{aligned} J_k^{\mu}\left(x_f\right) := {} & \left(x_k - \hat{x}_{k|k}\right)^T P_{k|k}^{-1}\left(x_k - \hat{x}_{k|k}\right) \\ & + w_k^T Q_k^{-1} w_k + v_{k+1}^T R_{k+1}^{-1} v_{k+1} \\ & + \mu\left(\|F_k x_k + G_k w_k - x_{k+1}\|^2\right. \\ & \left. + \|H_{k+1}x_{k+1} + K_{k+1}v_{k+1} - z_{k+1}\|^2\right), \end{aligned} \qquad (5)$$

whose solution, when $\mu \to +\infty$, approaches to the solution of the following constrained minimization problem:

$$\begin{aligned} & \min_{x_f} \left\{J_k(x_f)\right\} \\ & \text{s.t.} \begin{cases} x_{k+1} = F_k x_k + G_k w_k, \\ z_{k+1} = H_{k+1}x_{k+1} + K_{k+1}v_{k+1}, \end{cases} \end{aligned} \qquad (6)$$

with

$$J_k(x_f) = \left(x_k - \hat{x}_{k|k}\right)^T P_{k|k}^{-1}\left(x_k - \hat{x}_{k|k}\right) + w_k^T Q_k^{-1} w_k + v_{k+1}^T R_{k+1}^{-1} v_{k+1}.$$

It is shown in this paper that the use of penalty function method (Luenberger, 2003) is useful to design a recursive robust filter whose framework has a striking resemblance with the non-robust Kalman filter. In consequence, the convergence and stability analysis considered for the standard Kalman filter can be extended to the robust case.

3. Preliminary results

In this section we revisit the uncertain least-squares problem solved in Sayed and Nascimento (1999). We present the solution provided in this reference through an alternative array of matrices. In addition, we propose a framework based on the combination of the robust least-squares problem and the penalty function method (Luenberger, 2003) to deal with the optimal robust filtering problem.

Consider the following general optimization problem:

$$\min_{x} \max_{\delta A,\, \delta b} \left\{ \|x\|_{\mathcal{Q}}^2 + \|(A + \delta A)\,x - (b + \delta b)\|_W^2 \right\}, \qquad (7)$$

where A is a nominal matrix, b is a measurement vector, $\mathcal{Q} \succ 0$ and $W \succ 0$ are weighting matrices, x is an unknown vector, and δA and δb are perturbations modeled by

$$\begin{bmatrix}\delta A & \delta b\end{bmatrix} = H\Delta\begin{bmatrix}E_A & E_b\end{bmatrix}, \qquad \|\Delta\| \leq 1, \qquad (8)$$

with A, b, W, $\mathcal{Q}$, E_A, E_b, and H of appropriate dimensions and are assumed known. For estimation problems, the optimal solution $\hat{x}$ and the respective weighting matrix $\mathcal{P}$ for an estimation error $e = (x - \hat{x})$ can be rewritten in terms of an array of matrices

$$\begin{bmatrix}\hat{x} & \mathcal{P}\end{bmatrix} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 0 & I\end{bmatrix}$$
$$\times \begin{bmatrix} -\mathcal{Q} & 0 & 0 & I & 0 & 0 & 0 \\ 0 & -\hat{W} & 0 & 0 & I & 0 & 0 \\ 0 & 0 & -\hat{\lambda}I & 0 & 0 & I & 0 \\ I & 0 & 0 & 0 & 0 & 0 & I \\ 0 & I & 0 & 0 & 0 & 0 & A \\ 0 & 0 & I & 0 & 0 & 0 & E_A \\ 0 & 0 & 0 & I & A^T & E_A^T & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ b & 0 \\ E_b & 0 \\ 0 & -I \end{bmatrix}, \qquad (9)$$