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Brief paper

On reachable set estimation of singular systems☆

Zhiguang Feng[a,1], James Lam[b]

[a] College of Information Science and Technology, Bohai University, Jinzhou, Liaoning, 121013, China

[b] Department of Mechanical Engineering, The University of Hong Kong, Pokfulam Road, Hong Kong

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ABSTRACT

In this paper, the problem of reachable set estimation of singular systems is investigated. Based on the Lyapunov method, a sufficient condition is established in terms of a linear matrix inequality (LMI) to guarantee that the reachable set of singular system is bounded by the intersection of ellipsoids. Then the result is extended to the problem for singular systems with time-varying delay by utilizing the reciprocally convex approach. The effectiveness of the obtained results in this paper is illustrated by numerical examples.
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1. Introduction

Reachable set estimation of dynamic systems is to derive some closed bounded set to bound the set of all the states from the origin by inputs with bounded peak value. It is not only an important problem in robust control theory (Fridman & Shaked, 2003; Zuo, Ho, & Wang, 2010b), but also in practical engineering when safe operation is required through synthesizing controllers to avoid undesirable (or unsafe) regions in the state space (Hwang, Stipanovic, & Tomlin, 2003; Lygeros, Tomlin, & Sastry, 1999). In the latter context, the system is regarded safe if its reachable set does not contain any undesirable state. For example, suppose the velocity and the steering angle form the state of a vehicle and the state should be bounded in a set to avoid the vehicle from drifting and rolling over. Therefore, when the state lies within this set the vehicle can be operated safely, otherwise may be unsafe. Reachable set estimation of dynamic systems has various applications in peak-to-peak gain minimization (Abedor, Nagpal, & Poola, 1996), control systems with actuator saturation (Hu, Teel, & Zaccarian, 2006) and aircraft collision avoidance (Hwang et al., 2003). By using the S-procedure, the problem is investigated in Boyd, El Ghaoui, Feron, and Balakrishnan (1994) with the result derived in terms of linear matrix inequality (LMI) for the linear systems.

Time-delay, often attributed as one of the main causes of instability and performance degradation of a control system, has been extensively incorporated in models of many practical engineering systems, such as networked control systems, teleoperation and aircraft (Chiasson & Loiseau, 2007). For the reachable set estimation problem of time-delay systems, it is first solved in Fridman and Shaked (2003) based on the Lyapunov–Razumikhin method. The applications of reachable set estimation to disturbance rejection of time-delay systems (Cai, Huang, & Liu, 2010) and regional control of time-delay systems with saturating actuators (Fridman, Pila, & Shaked, 2003) are reported, respectively. An improved result is proposed in Kim (2008) by using the modified Lyapunov–Krasovskii type functional. By utilizing convex-hull properties in Kwon, Lee, and Park (2011) and constructing the maximal Lyapunov–Krasovskii functional in Zuo et al. (2010b), respectively, both results further improve that in Kim (2008). Very recently, the authors in Nam and Pathirana (2011) presented an improved bound of the reachable set using the delay partitioning method. When discrete and distributed delay appear simultaneously, the reachable set estimation problem is considered in Zuo, Fu, and Wang (2012).

Singular systems can better describe the behavior of some physical systems than state-space ones (Fridman, 2002; Lu, Ho, & Zhou, 2011; Zuo, Ho, & Wang, 2010a). Singular systems have been widely found in many practical systems, such as chemical processes, circuit systems, economic systems and aircraft modeling. Apart from their practical significance, they are of theoretical importance and have received a great deal of attention in recent

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E-mail addresses: congdian@gmail.com (Z. Feng), james.lam@hku.hk (J. Lam).

[1] Tel.: +86 0416 3400199.

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years owing to their fundamental differences from state-space systems. Many fundamental concepts and results based on the theory of state-space systems have been successfully extended to singular systems, such as stability and stabilization (Xu & Lam, 2004; Zhu, Zhang, & Feng, 2007), H_∞ control (Zhang, Xia, & Shi, 2008), model reduction (Xu & Lam, 2003), guaranteed cost control (Ren & Zhang, 2012) and dissipativity analysis (Wu, Park, Shu, & Chu, 2011). For nonlinear singular systems, there are several contributions on the reachable set estimation problem by employing different approaches. To mention a few, there are the level set method in Cross and Mitchell (2008), the Taylor models in Hoefkens, Berz, and Makino (2003) and Rauh, Brill, and Gunther (2009) and the differential inequalities in Scott and Barton (2013a,b). However, the conditions obtained in these works are difficult to solve or compute. On the other hand, time-varying delay is not considered in these nonlinear singular systems. Linear matrix inequalities (LMIs) can be solved efficiently via the Matlab LMI toolbox, Sedumi or Yalmip and LMI technique has been a powerful design tool in control theory and its applications (Boyd et al., 1994). For linear singular systems, some preliminary results about reachable set analysis are given in Feng and Lam (2014). For a class of nonlinearly affine singular systems, a state-feedback control approach is proposed in Azhmyakov, Poznyak, and Juarez (2013) by utilizing the LMI technique such that the state of closed-loop systems initiated in the ellipsoid remains inside the ellipsoid at all time instant.

In this paper, we extend the reachable set estimation result to singular systems. By using the Lyapunov–Krasovskii method, sufficient conditions are proposed in terms of LMIs and the intersection of ellipsoids is obtained to bound all states set of singular systems starting from the origin with a bounded input. Then the result is extended to singular systems with time-varying delay by utilizing reciprocally convex method. Finally, numerical examples are given to illustrate the effectiveness of the proposed results. There are major differences between this paper and (Azhmyakov et al., 2013). Firstly, the aim of Azhmyakov et al. (2013) is to find an ellipsoid (as small as possible) to bound the trajectory while our paper establishes a set (as small as possible but not necessarily an ellipsoid, it is the intersection of ellipsoids) to bound the state. Secondly, the system in Azhmyakov et al. (2013) is a nonlinear affine one while it is a linear one in this paper. The matrix before the exogenous disturbance in Azhmyakov et al. (2013) is the identity while it is a general matrix B in this paper. Furthermore, the singular system with time-varying delay is also considered in this paper. Thirdly, the regularity condition of matrix pair (E, A) is not included in the main results and regularity is assumed in Azhmyakov et al. (2013). However, the feasibility of results obtained in our paper can guarantee the regularity of the matrix pair.

The rest of this paper is briefly outlined as follows. In Section 2, the reachable set estimation problem of singular systems is formulated and solved. The reachable set bound of singular systems with time-varying delay is established in Section 3. Three illustrative examples are provided in Section 4 to show the effectiveness of our results. We conclude the paper in Section 5.

Notation: The notation used throughout the paper is standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $P > 0$ (≥ 0) means that P is real symmetric and positive definite (semi-definite); I and 0 refer to the identity matrix and zero matrix with compatible dimensions; $\star$ stands for the symmetric terms in a symmetric matrix and sym(A) is defined as $A+A^T$; $(M)_{m\times m}$ is the matrix composed of elements of first m rows and m columns of matrix M; $\|\cdot\|$ refers to the Euclidean vector norm and $x_t(\theta) = x(t+\theta)$ $(\theta \in [-\tau_M, 0])$. Matrices are assumed to be compatible for algebraic operations if their dimensions are not explicitly stated.

2. Reachable set estimation of singular system

Consider a class of linear continuous-time singular systems described by

$$\begin{cases} E\dot{x}(t) = Ax(t) + Bw(t) \\ x(0) \equiv 0, \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector; matrices E, A and B are constant matrices with appropriate dimensions and rank $(E) = n_1$; $w(t) \in \mathbb{R}^l$ represents a disturbance which satisfies

$$w^T(t)w(t) \leq \bar{w}^2 \tag{2}$$

where $\bar{w}$ is a real constant.

Before moving on, we give some definitions and lemmas which will be used in deriving the main results.

Definition 1 (*Xu & Lam, 2006*).

(1) The matrix pair (E, A) is said to be regular if $\det(sE - A)$ is not identically zero.
(2) The matrix pair (E, A) is said to be impulse free if $\deg\{\det(sE - A)\} = \operatorname{rank} E$.
(3) The matrix pair (E, A) is said to be stable if all the roots of $\det(sE - A)$ have negative real parts.
(4) The singular system in (1) is said to be admissible if it is regular ((E, A) is regular), impulse free ((E, A) is impulse free) and stable ((E, A) is stable).

Lemma 1 (*Xu & Lam, 2006*). *The matrix pair (E, A) is admissible if and only if there exists a matrix P such that*

$$E^TP = P^TE \geq 0, \qquad P^TA + A^TP < 0.$$

Lemma 2 (*Xu & Lam, 2006*). *If system (1) is regular and impulse free, there exist two non-singular matrices M and N such that*

$$MEN = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}, \qquad MAN = \begin{bmatrix} A_1 & 0 \\ 0 & I \end{bmatrix}.$$

A method to determine the transformation matrices M and N can be found in Algorithm 3.1 in Duan (2010). Let $\tilde{x}(t) = N^{-1}x(t) = \begin{bmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{bmatrix}$, where $\tilde{x}_1(t) \in \mathbb{R}^{n_1}$ and $\tilde{x}_2(t) \in \mathbb{R}^{n-n_1}$. Denote $MB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$. Then system (1) is restricted system equivalent to the following one:

$$\dot{\tilde{x}}_1(t) = A_1\tilde{x}_1(t) + B_1w(t) \tag{3}$$

$$0 = \tilde{x}_2(t) + B_2w(t). \tag{4}$$

Lemma 3 (*Boyd et al., 1994*). *Let $V(x(t))$ be a Lyapunov function for system (1)–(2) and $V(x(0)) = 0$. If $\dot{V} + \alpha V - \frac{\alpha}{\bar{w}^2}w^T(t)w(t) \leq 0$ with a scalar $\alpha > 0$, then $V(x(T)) \leq 1$ for $T \geq 0$.*

Proof. Denote

$$\dot{V}(x(t)) + \alpha V(x(t)) - \frac{\alpha}{\bar{w}^2}w^T(t)w(t) \leq 0. \tag{5}$$

Multiplying both sides of the inequality in (5) with $e^{\alpha t}$ yields

$$\begin{aligned} e^{\alpha t}\dot{V}(x(t)) + \alpha e^{\alpha t}V(x(t)) &= \frac{d}{dt}(e^{\alpha t}V(x(t))) \\ &\leq \frac{\alpha}{\bar{w}^2}e^{\alpha t}w^T(t)w(t). \end{aligned} \tag{6}$$

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