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Brief paper

Adaptive tracking control for a class of uncertain switched nonlinear systems ☆

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ABSTRACT

This paper is concerned with the problem of tracking control for a class of switched nonlinear systems in lower triangular form with unknown functions and arbitrary switchings. Two classes of state feedback controllers are constructed by adopting the adaptive backstepping technique, and both of them are designed by using the common Lyapunov function (CLF) method. The first controller is designed under multiple adaptive laws. Then, the second one is designed based on constructing a maximum common adaptive parameter, which can overcome the problem of over-parameterization of the first controllers. It is shown that the designed state-feedback controllers can ensure that all the signals remain bounded and the tracking error converges to a small neighborhood of the origin. Finally, simulation results are presented to show the effectiveness of the proposed approaches.

1. Introduction

In the past decades, switched systems have attracted much attention since it can be used to describe a large number of physical and engineering systems, such as networked control systems (Zhao, Hill, & Liu, 2009), near space vehicle control systems (Bao, Li, Chang, Niu, & Yu, 2010), circuit and power systems (Homaee, Zakariazadeh, & Jadid, 2014), to list a few. As the most important issues in the study of switched linear or nonlinear systems, stability analysis and control synthesis are discussed extensively by a lot of researchers, and many excellent results have been obtained for various types of switched systems under arbitrary switching or constraint switching; see e.g., Wang, Wang, and Shi (2009), Xiong, Lam, Gao, and Ho (2005), Zhang and Shi (2009), Zhao and Hill (2008a,b), Zhao, Liu, Yin, and Li (2014), Zhao, Yin, Li, and Niu (2014), Zhao, Zhang, Shi, and Liu (2012) and references therein.

It is well known that the stability of a switched system under arbitrary switching can be guaranteed if a CLF exists for all subsystems (Vu & Liberzon, 2005). Therefore, CLF has been extensively used for control synthesis of switched linear systems (Briat & Seuret, 2012, 2013; Lian & Zhao, 2010; Liberzon, 2003; Margaliot & Langholz, 2003; Xiang & Xiao, 2014). Recently, there have been some results reported on the global stabilization problem for switched nonlinear systems in strict-feedback form under arbitrary switchings by using the backstepping technique (Ma & Zhao, 2010; Wu, 2009). Meanwhile, Long and Zhao (2012) investigated the global stabilization problem for a class of switched nonlinear systems in the p-normal form by the so-called power integrator backstepping design method. Unfortunately, uncertainty is not taken into account in the aforementioned papers, which widely exists in practical switched nonlinear systems.

For general nonlinear systems in lower triangular form without switchings, it has been shown in the literature (Chen, Liu, Liu, & Lin, 2009; Krstić, Kokotović, & Kanellakopoulos, 1995) that the adaptive backstepping method is a powerful tool for controller design. As a base strategy in adaptive backstepping, the main objective is to cancel the unknown nonlinearity of system by constructing the virtual control functions and adaptive laws. This design method is of course expected to be useful to stabilize switched nonlinear systems with uncertainty. However, to the best of the authors' knowledge, there is not any related results on

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the stability analysis and controller design for uncertain switched nonlinear systems in lower triangular form under arbitrary switchings. The reason lies in that, if we want to construct a CLF, the common virtual control functions need to be found first, but it is somewhat difficult to find these common virtual control functions for different subsystems.

Based on the above illustrations, we know that the difficulty encountered in constructing a CLF is how to design a common virtual control function at each step when the adaptive backstepping technique is applied to switched nonlinear systems. One way to deal with this problem is to simply assume the existence and availability of a common virtual control function as was done in Wu (2009). Most recent, the authors in Ma and Zhao (2010) proposed some inequality constraints on the common virtual functions to improve previous results. However, it is required in Ma and Zhao (2010) that the common virtual functions should be first-order derivative, not related to switching laws, and satisfy some inequality constraints, which is somewhat too strict in practice to be applied. Furthermore, the uncertainty is not considered in Ma and Zhao (2010) and Wu (2009). Therefore, the design of the common virtual functions has not been fully solved in the existing literature when the system contains uncertainty, which motivates our present work.

In this paper, the adaptive tracking problem is studied for a class of switched nonlinear systems with completely unknown uncertainties. An efficient approach of constructing common virtual control functions is proposed for the considered systems, and state-feedback controllers are designed via the adaptive backstepping technique where a Mamdani-type fuzzy logic system is utilized to approximate the redefined unknown functions. A design method with multiple adaptive laws is presented in first place. Furthermore, another method with only one adaptive law is proposed to avoid the problem of over-parameterization. Both controllers can guarantee that all the closed-loop signals remain bounded, and the system output tracks the reference signal while the computation burden is low.

Notations: in this paper, the notations are standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space, the notation $\|\cdot\|$ refers to the Euclidean vector norm. For positive integers $1 \le i \le n$, $1 \le j \le m$, we also denote $\Xi_{i,\max} = \max\{\Xi_{i,j} : 1 \le j \le m\}$, $\Xi_{i,\min} = \min\{\Xi_{i,j} : 1 \le j \le m\}$.

2. Problem formulation and preliminaries

Consider a class of switched nonlinear system in the following form:

$$\begin{aligned} \dot{x}_i &= g_{i,\sigma(t)}x_{i+1} + f_{i,\sigma(t)}(\bar{x}_i), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n &= g_{n,\sigma(t)}u_{\sigma(t)} + f_{n,\sigma(t)}(\bar{x}_n), \\ y &= x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i := (x_1, x_2, \ldots, x_i)^T \in \mathbb{R}^i$, $i = 1, 2, \ldots, n$ is the system state, y is the system output; $\sigma(t) : [0, +\infty) \to M = \{1, 2, \ldots, m\}$ is the switching signal; $u_k \in R$ is the control input of the kth subsystem; For any $i = 1, 2 \ldots, n$ and $k = 1, 2, \ldots, m$, $f_{i,k}(\bar{x}_i)$ is an unknown smooth nonlinear function representing the system uncertainty, and $g_{i,k}$ is a positive constant.

Our control objective is to design state-feedback controllers such that the output of system (1) tracks a given time-varying signal $y_d(t)$ within a bounded error and all the signals of the closed-loop systems remain bounded under arbitrary switchings.

Assumption 1. The tracking target $y_d(t)$ and its time derivatives up to the nth order are continuous and bounded.

Remark 1. System (1) will be reduced to system (1) in Ma and Zhao (2010) when the tracking control problem and the uncertain functions are not taken into account. Therefore, the systems considered in this paper are more general.

In the controller design and stability analysis procedure, fuzzy logic systems will be used to approximate the unknown functions. Therefore, the following useful concept and lemma are first recalled.

Fuzzy logic systems include some IF–THEN rules, and the ith IF–THEN rule is written as

R_i : If x_1 is F_1^i and ... and x_n is F_n^i then y is B^i,

where $x = [x_1, x_2, \ldots, x_n]^T \in \mathbb{R}^n$, and $y \in \mathbb{R}$ are input and output of the fuzzy logic systems, respectively. $F_1^i, F_2^i, \ldots, F_n^i$ and B^i are fuzzy sets in $\mathbb{R}$. By using the strategy of singleton fuzzification, the product inference and the center-average defuzzification, the fuzzy logic system can be formulated as

$$y(x) = \frac{\sum_{i=1}^{N} w_i \prod_{j=1}^{n} \mu_{F_j^l}(x_j)}{\sum_{i=1}^{N}\left[\prod_{j=1}^{n} \mu_{F_j^l}(x_j)\right]},$$

where N is the number of IF–THEN rules, w_i is the point at which fuzzy membership function $\mu_{B^i}(w_i) = 1$. Let $s_i(x) = \prod_{j=1}^{n} \mu_{F_j^l}(x_j) / \sum_{i=1}^{N}[\prod_{j=1}^{n} \mu_{F_j^l}(x_j)]$, $S(x) = [s_1(x), \ldots, s_N(x)]^T$ and $W = [w_1, w_2, \ldots, w_N]^T$. Then the fuzzy logic system can be rewritten as

$$y = W^T S(x), \tag{2}$$

If all memberships are chosen as Gaussian functions, the following lemma holds.

Lemma 1 (*Wang & Mendel, 1992*). *Let $f(x)$ be a continuous function defined on a compact set Ω. Then, for a given desired level of accuracy $\varepsilon > 0$, there exists a fuzzy logic system (2) such that*

$$\sup_{x \in \Omega} \left| f(x) - W^T S(x) \right| \le \varepsilon.$$

Remark 2. Lemma 1 plays a key role in the following design procedure and it indicates that any given real continuous function $f(x)$ can be represented by the linear combination of the basis function vector $S(x)$ within a bounded error ε. That is, $f(x) = W^T S(x) + \delta(\varepsilon)$, $|\delta(\varepsilon)| \le \varepsilon$. It is noted that $0 < S^T S \le 1$.

3. Main Results

In this section, we will present adaptive fuzzy control scheme for system (1) via the backstepping technique. In Section 3.1, a detailed design procedure will be given. In each step, a common virtual control function α_i should be designed by using an appropriate common Lyapunov function V_i, and the control law u_k will finally be designed. To avoid repetition, in Section 3.2, we only adopt a final common Lyapunov function to demonstrate the design procedure.

3.1. Adaptive control design under multiple adaptive laws

In this subsection, a systemic control design procedure under multiple adaptive laws will be presented. Design the control laws as

$$u_k = -\frac{1}{g_{n,k}}\left(\frac{\hat{\theta}_n}{2\zeta_{n,\min}^2} z_n + \lambda_n z_n + \frac{z_n}{2}\right), \tag{3}$$

where $\zeta_{n,k}$ and λ_n are positive design parameters, $\zeta_{n,\min} = \min\{\zeta_{n,k} : k \in M\}$, $\hat{\theta}_n$ is the estimation of $\theta_n = \left\|W_{n,\max}\right\|^2$, $W_{n,\max} = \max\{W_{n,k} : k \in M\}$ and $W_{n,k}$ is used in fuzzy logic system $W_{n,k}^T S_{n,k}$