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Cramer–Rao type bounds for sparsity-aware multi-sensor multi-target tracking

Saurav Subedi[a,1], Yimin D. Zhang[b,1,*], Moeness G. Amin[a,1], Braham Himed[c]

[a] The Center for Advanced Communications, Villanova University, Villanova, PA 19085, USA
[b] Department of Electrical and Computer Engineering, Temple University, Philadelphia, PA 19122, USA
[c] RF Technology Branch, Air Force Research Laboratory, AFRL/RYMD, Dayton, OH 45433, USA

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ABSTRACT

Conventionally, sparsity-aware multi-sensor multi-target tracking (MTT) algorithms comprise a two-step architecture that cascades group sparse reconstruction and MTT algorithms. The group sparse reconstruction algorithm exploits the *a priori* information that the measurements across multiple sensors share a common sparse support in a discretized target state space and provides a computationally efficient technique for centralized multi-sensor information fusion. In the succeeding step, the MTT filter performs the data association, compensates for the missed detections, removes the clutter components, and improves the accuracy of multi-target state estimates according to the pre-defined target dynamic model. In a recent work, a novel technique was proposed for sparsity-aware multi-sensor MTT that deploys a recursive feedback mechanism such that the group sparse reconstruction algorithm also benefits from the *a priori* knowledge about the target dynamics. As such, it is of significant interest to compare the tracking performance of these methods to the optimal multi-sensor MTT solution, with and without considering the missing samples. In this paper, we analytically evaluate the Cramer-Rao type performance bounds for these two schemes for sparsity-aware MTT algorithms and show that the recursive learning structure outperforms the conventional approach, when the measurement vectors are corrupted by missing samples and additive noise.

1. Introduction

In recent years, sparsity-aware multi-target tracking (MTT) algorithms have attracted significant research interest. Researchers have proposed several techniques (e.g., [2–5]) to exploit the *a priori* knowledge that there is only a small number of targets to be tracked over a pre-defined surveillance area, and hence, the measurements are sparse, either in their natural basis or some other sparsifying basis. Recently, the hierarchical Kalman filter has been proposed in [6] to track the dynamic sparse signals, which incorporates the fundamentals of sparse Bayesian learning into the traditional Kalman filtering, where the output of the tracking filter is exploited to update the covariance matrix of the process noise, thereby enforcing sparsity constraint into the traditional Kalman filtering framework.

Conventionally, the sparsity-aware MTT algorithm cascades the sparse signal reconstruction algorithm and the multi-target tracking algorithm in succession. First, the sparse reconstruction algorithm is exploited to estimate the multi-target state, and in the succeeding step, multi-target state estimates are fed as inputs to the MTT filters for data association, clutter removal, compensation for missed detections, and reduction in the localization error. For multi-sensor MTT [7], group sparse reconstruction algorithms have been deployed as computationally efficient techniques for a centralized multi-sensor information fusion. The *a priori* information that the measurements across multiple sensors share a common sparse support in a discretized target state space allows for the exploitation of the group sparse reconstruction. As such, the output of the group sparse reconstruction algorithm obtained in the form of instantaneous estimates of the multi-target states is fed as the input to the MTT filters. The overall performance of these techniques relies on the ability of the group sparse reconstruction algorithm to accurately and reliably estimate the instantaneous multi-target states.

* Corresponding author.
E-mail address: ydzhang@temple.edu (Y.D. Zhang).

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In many practical applications, the observations suffer from a high proportion of missing samples, due to fading, shadowing or removal of impulsive noise, and is corrupted by a strong additive noise, rendering it difficult to accurately estimate the multi-target states using group sparse reconstruction-based methods. Recently, a novel technique is proposed in [8] for sparsity-aware multi-sensor MTT that deploys a recursive feedback mechanism such that the group sparse reconstruction algorithm and the conventional MTT filter interplay and learn from each other. Such recursive learning approach creates a global learning architecture that enables the group sparse reconstruction algorithm to benefit from the *a priori* knowledge about the target dynamics. Numerical results presented in [8] in terms of the optimal sub-pattern assignment (OSPA) metric [9] show that the methods proposed therein enable a significant performance improvement over the conventional approach through such a feedback mechanism. This is particularly evident when the measurement vectors comprise a high percentage of missing samples and are corrupted by strong additive noise.

The unconditional posterior Cramer–Rao lower bound (PCRLB) [10] provides a theoretical performance limit of any estimator for a non-linear filtering problem under the Bayesian framework. In [11], the authors derived a recursive approach to calculate the sequential PCRLB for a general multi-dimensional discrete-time non-linear filtering problem. Several variants of the PCRLB have been proposed in the literature to make the PCRLB more adaptive. For instance, in [12–14], the PCRLB is conditioned on the measurements up to a reset initial time in lieu of the absolute initial state as in the vanilla PCRLB definition. Instead of representing the posterior probability density function of the system state at the reset initial time non-parametrically by a set of random particles as in [12], a systematic recursive approach is used to derive the exact conditional PCRLB based on first principles in [15]. Two other online conditional PCRLBs are proposed in [16] as alternatives to the one proposed in [15], and are shown to provide similar results through numerical examples. These variants have rendered the prior knowledge of the initial system state more useful and relevant in the PCRLB evaluation, particularly in situations when the state process noise is high and thus the prior knowledge regarding the system state at the initial time quickly becomes irrelevant. However, to the best of our knowledge, none of the existing works provide a conditional PCRLB for situations where the measurement is unreliable due to strong additive noise and/or a high proportion of missing samples. As such, the existing literature still lacks the PCRLB analysis for sparsity-aware multi-sensor MTT problems.

In this paper, we analytically evaluate the performance bounds, for the two aforementioned architectures for sparsity-aware multi-sensor MTT, namely, the conventional architecture and the global learning architecture. We quantify the degradation in the overall tracking performance when the measurement vectors suffer from a high percentage of missing samples and strong additive noise. First, we derive the performance bounds for the estimation of the instantaneous multi-target state exploiting the group sparse signal reconstruction algorithm in the case of a signal model comprising missing samples and additive white Gaussian noise perturbation [17]. Assuming an optimal estimation of the instantaneous multi-target state by the group sparse reconstruction algorithm under the given signal conditions, we evaluate the performance bound for the MTT algorithm. Next, we analytically evaluate the performance improvement achieved by implementing the recursive learning architecture, where the *a priori* knowledge about the target dynamics is exploited at the sparse reconstruction stage through a feedback mechanism. To summarize, the key contributions of this paper are follows:

1. We analytically quantify the effect of missing samples and additive noise on the performance of sparsity-aware multi-sensor MTT algorithms.
2. We evaluate the limits on the performance improvement that can be achieved by implementing the recursive learning architecture assuming optimal estimation at both stages - group sparse reconstruction and MTT.
3. We analyze the boundary conditions for which the recursive learning architecture guarantees a convergence and assess the effect of relative weight on the achievable performance improvement.

The remainder of the paper is organized as follows. Section 2 describes the target dynamic model, presents the signal model, considering the effect of missing samples and additive white Gaussian noise. Section 3 presents a high-level overview of the two approaches for sparsity-aware MTT. Section 4 presents the analytical comparison of the performance bounds for these two approaches. Section 5 provides simulation results in the case of a multi-target tracking in a multi-static passive Doppler sensor network, and finally conclusions are drawn in Section 6.

Notations: A lower (upper) case bold letter denotes a vector (matrix). Specifically, $\mathbf{I}_N$ and $\mathbf{0}_N$ denote the $N \times N$ identity and zero matrices, respectively. $(\cdot)^T$ and $(\cdot)^H$, respectively, denote transpose and Hermitian operations, and $\circ$ denotes the Hadamard product. diag$(\cdot)$ forms a diagonal matrix from a vector, tr$(\cdot)$ stands for matrix trace, and Re$(\cdot)$ denotes the real part of a complex variable. E$(\cdot)$ stands for the expectation operation. $\mathbb{C}^{m\times n}$ and $\mathbb{C}^{m\times 1}$ represent an $m \times n$-dimensional complex matrix and an m-element complex vector, respectively. Likewise, $\mathbb{R}^{m\times 1}$ represents an m-element real vector. $\|\cdot\|_n$ denotes the l_n-norm of a vector, and $x \sim \mathcal{N}(a, b)$ and $x \sim \mathcal{CN}(a, b)$, respectively, denote variable x to be real and complex Gaussian distributed with mean a and variance b.

2. Signal model

2.1. Target dynamics

We consider the problem of tracking K moving targets, where K is unknown. The ground truth state vector associated with the kth target at the tth observation instant is represented as $\boldsymbol{\theta}_{t,k} \in \mathbb{R}^{D\times 1}$, for $k = 1, \cdots, K$ and $t = 1, \cdots, T$. Herein, we refer to the observation instants as the time instants at which the sensors report their measurement vectors to the fusion center. Note that each measurement comprises several discrete-time samples of the waveform received at the sensor. The number of samples per measurement vector depends on the observation interval and the sampling rate deployed at the sensor. At each observation instant, the ground truth state set is defined as $\boldsymbol{\Theta}_t \triangleq [\boldsymbol{\theta}_{1,t}^T, \cdots, \boldsymbol{\theta}_{K,t}^T]^T$. The target dynamics is assumed to evolve according to a linear Gaussian model, such that

$$\boldsymbol{\theta}_{t,k} = \mathbf{F}\boldsymbol{\theta}_{t-1,k} + \mathbf{w}_{t,k}, \tag{1}$$

where $\mathbf{F}$ is the state transition matrix and $\mathbf{w}_{t,k} \sim \mathcal{N}(\mathbf{0}, \mathbf{Q})$ is the process noise modeled as additive white Gaussian. The definitions of the state transition matrix $\mathbf{F}$ and the covariance matrix of the process noise $\mathbf{Q}$ depend on the application. Application examples will be provided in Section 5.

2.2. Observation with missing samples

We consider R receivers monitoring the region of interest. The multi-target states are represented as a multi-component signal in the observation space at the rth receiver through a deterministic