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Brief paper

Stabilization of neutral time-delay systems with actuator saturation via auxiliary time-delay feedback ☆

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ABSTRACT

This paper investigates the stabilization problem for neutral time-delay systems with actuator saturation. Different from the existing techniques, the auxiliary time-delay feedback is introduced for the first time in this paper. Based on such a technique, the saturation nonlinearity is represented as the convex combination of state feedback and auxiliary time-delay feedback. By employing free-weighting matrix technique and Jensen integral inequalities, and performing the accurate estimation of the lower bounds of L–K functionals, the improved delay-dependent local stabilization conditions are proposed in terms of linear matrix inequalities (LMIs). Numerical examples illustrate the reduced conservatism of the proposed conditions in this paper.

1. Introduction

Time-delays are frequently encountered in various practical systems, such as chemical engineering systems, biological systems and manufacturing processes (Gu, Kharitonov, & Chen, 2003). There are two types of time-delay systems, i.e., retarded type and neutral type. The retarded type contains delays only in its states, while the neutral type contains delays in both its states and its derivatives of the states. On the other hand, it is well recognized that LMI-based approaches are more convenient for solving corresponding synthesis problems, and delay-dependent results are generally less conservative than delay-independent ones especially when the size of delay is small (Xu & Jam, 2008). During the past two decades, several important techniques have been proposed to obtain LMI-based delay-dependent analysis and synthesis conditions for time-delay systems, see, e.g., Chen and Zheng (2007), Fridman (2001), Han (2009), He, Wang, Lin, and Wu (2005); He, Wang, Xie, and Lin (2007); He, Wu, She, and Liu (2004), Li, Jing, and Karimi (2014), Qian, Liu, and Fei (2012) and Sun, Liu, and Chen (2009).

In many practical control applications, actuator saturation is often inevitable, and its existence may deteriorate the performance of a control system and even cause the instability of closed-loop system. Therefore, considerable attention has been devoted to linear systems subject to saturating controllers during the past decades, see e.g., Alamo, Cepeda, and Limon (2005), Gomes da Silva and Tarbouriech (2005); Hu and Lin (2001), Hu, Lin, and Chen (2002), Lin (1998), Tarbouriech, Garcia, Gomes da Silva, and Queinnec (2011), Zhou (2013) and Zhou, Lin, and Duan (2008). Generally speaking, the current research can be classified into two categories according to whether the open-loop poles are located on the closed left-half plane, i.e., global/semi-global stabilization, and local stabilization and anti-windup design. For the local stabilization and anti-windup design, two dominant approaches are proposed to deal with the saturation nonlinearity, one is the polytopic models (Alamo et al., 2005; Hu & Lin, 2001; Hu et al., 2002; Zhou, 2013) and the other is the generalized sector condition (Gomes da Silva & Tarbouriech, 2005). In particular, it is worth mentioning that the saturation representation proposed in Alamo et al. (2005) and Tarbouriech et al. (2011) with the compact notation proposed in Zhou (2013) contains more slack variables, and thus is less conservative than that in Hu and Lin (2001) and Hu et al. (2002) for the multiple input systems.

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For the systems with both time-delay and actuator saturation, global/semi-global stabilization were well investigated in Lin and Fang (2007), Yakoubi and Chitour (2007), Zhou, Lin, and Duan (2010) and Zhou, Lin, and Duan (2012) under the assumption that the open-loop poles are located on the closed left-half plane. Removing such a restriction on open-loop poles, the problem of local stabilization has been widely studied in Cao, Lin, and Hu (2002), Fridman, Pila, and Shaked (2003), Gomes da Silva, Seuret, Fridman, and Richard (2005, 2011), Tarbouriech and Gomes da Silva (2000) and Zhang, Boukas, and Haidar (2008) by incorporating the techniques of analyzing the stability of time-delay systems. However, it should be pointed out that the techniques of representing saturation nonlinearity in the above references are the same as the cases without delay. It is clear that the time-delay information is completely neglected when dealing with the saturation nonlinearity, which may result in some conservative results.

In this paper, we resist the stabilization problem for neutral systems with time-varying delay and actuator saturation. Different from the existing techniques, the auxiliary time-delay feedback is proposed in this paper. Based on the polytopic approach proposed in Alamo et al. (2005), Tarbouriech et al. (2011) and Zhou (2013), the saturation nonlinearly is firstly represented by the convex combination of state feedback and auxiliary time-delay feedback. By incorporating Lyapunov–Krasovskii (L–K) functional theory, free-weighting matrix technique and integral inequalities, and performing the accurate estimation of the lower bounds of L–K functionals, then the improved stabilization conditions are obtained in terms of LMIs. Compared with the existing results, the main novelty of this paper is that the auxiliary time-delay feedback is introduced for the first time when representing the saturation nonlinearity, and the accurate estimation of the lower bounds of L–K functionals is performed to obtain LMI-based conditions. Finally, the reduced conservatism of the proposed conditions in this paper is shown by numerical examples.

Notation. $\lambda_M(P)$ denotes the maximum eigenvalue of matrix P. A real symmetric matrix $P > 0 (\geq 0)$ denotes P being a positive definite (positive semi-definite) matrix. I denotes an identity matrix with proper dimension. Matrices, if not explicitly stated, are assumed to have compatible dimensions. The space of the continuously differentiable vector functions ϕ over $[-h, 0]$ is denoted by $C^1[-h, 0]$. $\|\cdot\|$ and $\|\cdot\|_\infty$ denote the 2-norm and ∞-norm, respectively, and $\max_{t\in[-h,0]} \|\phi(t)\|$ is denoted by $\|\phi\|_c$. $\mathbb{I}[1, \pi]$ denotes the set of integers whose elements are $1, 2, \ldots, \pi$. $\mathbb{D}_m$ denotes the set of $m \times m$ diagonal matrices with diagonal elements either 1 or 0. $e_{m,k} \in \mathbb{R}^{1\times m}$ denotes a row vector whose k-th element is 1 and the others are zero, and $\otimes$ denotes the Kronecker product.

2. Problem formulation

Consider the following neutral time-delay system with actuator saturation

$$\dot{x}(t) - C\dot{x}(t-h(t)) = Ax(t) + A_d x(t-h(t)) + Bsat(u(t)), \quad (1)$$

$$x(t) = \phi(t), \quad \forall t \in [-h, 0], \quad (2)$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, A, A_d, B and C are known real constant matrices with appropriate dimensions, $h(t)$ denotes time-varying delay that satisfies $0 \leq h(t) \leq h$ and $\dot{h}(t) \leq \mu < 1$. $sat(u) : \mathbb{R}^m \to \mathbb{R}^m$ is the vector valued standard saturation function described by $sat(u) = [sat(u_1) \quad sat(u_2) \cdots sat(u_m)]^T$, where $sat(u_j) = sgn(u_j) \min\{1, |u_j|\}$, $j \in \mathbb{I}[1, m]$. The controller used in this paper is the following state feedback

$$u(t) = Kx(t), \quad (3)$$

where $K \in \mathbb{R}^{m\times n}$ is the gain matrix to be designed.

In this paper, it is assumed that $\phi(t)$ is continuously differentiable over $[-h, 0]$, and one of our interests is to estimate the domain of attraction of the following form

$$X_\rho = \left\{\phi(t) \in C^1[-h, 0] : \|\phi\|_c \leq \rho_1, \|\dot{\phi}\|_c \leq \rho_2\right\}, \quad (4)$$

where ρ_1 and ρ_2 are some scalars to be maximized.

Lemma 1 (Zhou (2013)). *Let $m \geq 1$ be a given integer, and $v \in \mathbb{R}^{\overleftrightarrow{m}}$ be such that $\|v\|_\infty \leq 1$, where $\overleftrightarrow{m} = m2^{m-1}$. Let the elements in $\mathbb{D}_m$ be labeled as D_i, $i \in \mathbb{I}[1, 2^m]$, and the function f_m be defined as $f_m(0) = 0$ and*

$$f_m(i) = \begin{cases} f_m(i-1) + 1, & D_i + D_j \neq I_m, \quad \forall j \in \mathbb{I}[1, i] \\ f_m(j), & D_i + D_j = I_m, \quad \exists j \in \mathbb{I}[1, i]. \end{cases}$$

Then for any $u \in \mathbb{R}^m$, there holds

$$sat(u) \in co\left\{D_i u + \mathcal{D}_i^- v : i \in \mathbb{I}[1, 2^m]\right\},$$

where "co" denotes the convex hull, and $\mathcal{D}_i^- \in \mathbb{R}^{m\times\overleftrightarrow{m}}$ is defined as $\mathcal{D}_i^- = e_{2^{m-1}, f_m(i)} \otimes D_i^-$ with $D_i^- = I - D_i$.

Assume that there exist matrices $U \in \mathbb{R}^{\overleftrightarrow{m}\times n}$, $V \in \mathbb{R}^{\overleftrightarrow{m}\times n}$ and $W \in \mathbb{R}^{\overleftrightarrow{m}\times n}$ such that the restrictive condition $\|v(t)\|_\infty = \|Ux(t) + Vx(t-h(t)) + Wx(t-h)\|_\infty \leq 1$ holds for $t \geq 0$, then it follows from (1) and (3), and Lemma 1 that the closed-loop system can be written as

$$\begin{aligned} \dot{x}(t) = & \sum_{i=1}^{2^m} \lambda_i(t)\Big\{\big[A + B(D_i K + \mathcal{D}_i^- U)\big]x(t) \\ & + (A_d + B\mathcal{D}_i^- V)x(t-h(t)) \\ & + B\mathcal{D}_i^- Wx(t-h) + C\dot{x}(t-h(t))\Big\} \triangleq \chi(t), \end{aligned} \quad (5)$$

where $\lambda_1(t) \geq 0, \ldots, \lambda_{2^m}(t) \geq 0$ and $\sum_{i=1}^{2^m} \lambda_i(t) = 1$.

Remark 1. In Cao et al. (2002), Fridman et al. (2003) and Zhang et al. (2008), the auxiliary feedback of the from $v(t) = Hx(t)$ was introduced under the assumption that $|h_l x(t)| \leq \bar{u}_l$, $l \in [1, m]$. Different from the techniques in Cao et al. (2002), Fridman et al. (2003) and Zhang et al. (2008), the auxiliary time-delay feedback $v(t) = Ux(t) + Vx(t-h(t)) + Wx(t-h)$ is introduced in this paper. Compared with some existing results, our proposed stabilization conditions will be more slack due to the introduction of time-delay feedback matrices V and W, and thus the larger estimates of the domain of attraction can be obtained by the conditions in this paper.

3. Main results

In this section, we will establish the improved local stabilization conditions in terms of LMIs.

Theorem 1. *For given scalars $h, \mu < 1$ and $\delta \neq 0$, if there exist symmetric matrices $\bar{P} > 0$, $\bar{Q}_1 = \begin{bmatrix} \bar{Q}_{11} & \bar{Q}_{12} \\ \bar{Q}_{12}^T & \bar{Q}_{13} \end{bmatrix} > 0$, $\bar{Q}_2 = \begin{bmatrix} \bar{Q}_{21} & \bar{Q}_{22} \\ \bar{Q}_{22}^T & \bar{Q}_{23} \end{bmatrix} > 0$, $\bar{Z} > 0$, and any matrices $X, Y, G, H, L, \bar{M}_k, \bar{N}_k$, k*