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Brief paper

# Exact tracking control of nonlinear systems with time delays and dead-zone input[☆]

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ABSTRACT

This paper is concerned with the control design problem for a class of nonlinear systems with the state time-varying time delays and nonsymmetric dead zone. The problem addressed is to design adaptive controllers that guarantee the exact tracking of a given reference signal. Two continuous robust adaptive control schemes are proposed. A positive nonlinear control gain function, which is not required to satisfy an inequality but is expressed explicitly, is carefully constructed and is used in the control law and adaptive law. An illustrative example is provided to demonstrate the validity of the proposed design method.



## 1. Introduction

In the last two decades, the problem of adaptive control for nonlinear systems with time delays or dead-zone input has been extensively studied (see, e.g. Chen, Liu, Liu, & Lin, 2009; Ge & Tee, 2007; Ibrir, Xie, & Su, 2007; Karafyllis & Krstic, 2013; Tao & Kokotovic, 1994; Tong & Li, 2012; Wu, 2009; Yoo, Park, & Choi, 2009; Zhou, Wen, & Zhang, 2006; Zhang, Xu, & Zhang, 2014; respectively). Recently, adaptive control for nonlinear systems with both time delays and dead-zone inputs has received much attention. In Shyu, Liu, and Hsu (2005), a decentralized variable structure controller was designed for a class of uncertain large scale systems with time delay in the interconnection and dead-zone nonlinearity in the input. The work in Zhang and Ge (2007), which considered a class of multi-input multi-output nonlinear state delay systems in a triangular structure with unknown nonlinear dead-zones and gain signs, proposed a neural network approximation based adaptive control scheme. In Yoo (2010) and Zhou (2008), the backstepping technique based decentralized adaptive control problems were addressed for interconnected time-delay nonlinear systems with the input of each loop preceded by an unknown dead zone, where adaptive control laws with and without dead-zone inverse were developed, respectively. The tracking control problem for a class of nonlinear systems with time delays and dead-zone input was investigated in Hua, Wang, and Guan (2008), where a smooth adaptive controller was constructed.

However, except for Shyu et al. (2005), the presented adaptive controllers do not produce the exact tracking in the presence of the time delay and dead-zone input. On the other hand, although the perfect tracking is achieved in Shyu et al. (2005), the discontinuous control law may cause chattering at certain boundaries with adverse effects on performance. In this paper, we address the adaptive control design problem for a class of nonlinear systems with time-varying delays and nonsymmetric dead zone in the actuator. The main contributions of the paper are listed as follows: (i) exact tracking is achieved in the sense that the tracking error converges to zero asymptotically or exponentially; (ii) the explicit expression for the control gain function $\rho(\chi)$, $\chi \geq 0$, is provided.

## 2. Problem statement

Consider the nonlinear system in the following form:

$$\begin{aligned} \dot{x}_i(t) &= x_{i+1}(t), \quad i = 1, 2, \ldots, n-1, \\ \dot{x}_n(t) &= f(t, x_1(t-d_1(t)), x_2(t-d_2(t)), \ldots, \\ &\quad x_n(t-d_n(t))) + \Gamma(u(t)), \end{aligned} \tag{1}$$

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where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in R^n$ is the state variable, $u(t) \in R$ is the control input, $f(\cdot)$ represents the delayed state perturbation, $d_i(t)$ is the time-varying time delay satisfying $0 \le d_i(t) \le d_i^*, \dot{d}_i(t) \le \bar{d}_i < 1$, $i = 1, 2, \ldots, n$, $x(t) = \varphi(t), t \in [-d^*, 0]$, $\varphi(t)$ is the initial condition, $d^* = \max\{d_1^*, d_2^*, \ldots, d_n^*\}$, and $\Gamma(u(t))$ denotes the nonsymmetric dead-zone input, which is defined by

$$\Gamma(u(t)) = \begin{cases} m_r(u(t) - b_r), & \text{if } u(t) \ge b_r, \\ 0, & \text{if } -b_l < u(t) < b_r, \\ m_l(u(t) + b_l), & \text{if } u(t) \le -b_l, \end{cases} \tag{2}$$

with positive left and right slopes $m_l, m_r$ and positive breakpoints $b_l, b_r$.

The control objective is to design the memoryless adaptive controllers for the system (1) such that the plant state $x(t)$ exactly tracks a given reference signal $x_d(t) = [y_d(t), \dot{y}_d(t), \ldots, y_d^{(n-1)}(t)]^T$, while all the closed-loop signals remain bounded, where $y_d(t)$ and its first $n$ derivatives are known and bounded.

**Assumption 1.** For the uncertain function $f(\cdot)$, there exist known and continuously differentiable class-$K$ functions $\alpha_i(\chi)$, $\chi \ge 0$, $i = 1, 2, \ldots, n$, and unknown positive constants $\gamma, \theta_i$, $i = 1, 2, \ldots, n$, such that the following inequality holds: $|f(t, x_1(t - d_1(t)), x_2(t - d_2(t)), \ldots, x_n(t - d_n(t)))| \le \sum_{i=1}^n \theta_i \alpha_i(|x_i(t - d_i(t))|) + \gamma$.

**Assumption 2.** The parameters $d_i^*, \bar{d}_i, i = 1, 2, \ldots, n$, and $m_l, m_r, b_l, b_r$, are unknown.

## 3. Adaptive control design and analysis

We first define the tracking error as $E_i(t) = x_i(t) - y_d^{(i-1)}(t) = x_i(t) - x_{d_i}(t)$, $E(t) = [E_1(t), E_2(t), \ldots, E_n(t)]^T$. From (1), the dynamics of the tracking error is

$$\dot{E}(t) = AE(t) + B[f + \Gamma(u(t)) - y_d^{(n)}(t)], \tag{3}$$

where $A = \begin{bmatrix} 0 & I_{n-1} \\ 0 & 0 \end{bmatrix}_{n\times n}$, $B = \begin{bmatrix} 0_{n-1} \\ 1 \end{bmatrix}_{n\times 1}$. By this, we can choose a gain matrix $K$ such that the matrix $A + BK$ is stable. Thus, the matrix $P > 0$ exists and is the solution to the Lyapunov equation $P(A + BK) + (A + BK)^T P = -Q$ with any $Q > 0$.

### 3.1. Scheme I: Asymptotic tracking

According to the definition of class-$K$ function, there exists a function $\bar{\alpha}_i(\cdot)$ such that $\alpha_i(\chi) = \chi\bar{\alpha}_i(\chi)$, $\chi \ge 0$. Then, we propose the following adaptive controller:

$$\begin{aligned} u(t) &= -\frac{1}{2}\hat{\theta}_1(t)\rho(W(t))E^T(t)PB \\ &\quad - \frac{\hat{\gamma}_1^2(t)\rho(W(t))E^T(t)PB}{2\hat{\gamma}_1(t)\rho(W(t))|E^T(t)PB| + 2\sigma_1(t)}, \end{aligned} \tag{4}$$

$$\dot{\hat{\theta}}_1(t) = l_{11}\rho^2(W(t))(E^T(t)PB)^2 - l_{11}\sigma_2(t)\hat{\theta}_1(t), \tag{5}$$

$$\dot{\hat{\gamma}}_1(t) = l_{12}\rho(W(t))|E^T(t)PB| - l_{12}\sigma_3(t)\hat{\gamma}_1(t), \tag{6}$$

where $W(t) = E^T(t)PE(t)$, $\rho(\chi) = c_1 + c_2\sum_{i=1}^n \bar{\alpha}_i^2\left(2\sqrt{\frac{\chi}{\lambda_{\min}(P)}}\right)$, $\chi \ge 0$, $c_1, c_2$ are positive design constants, $\lambda_{\min}(\cdot)$ denotes the minimum eigenvalue of a matrix, $\hat{\theta}_1(t), \hat{\gamma}_1(t)$ are respectively the estimates of $\theta_1^*, \gamma^*$, which are defined below (10), $\hat{\theta}_1(0) \ge 0$, $\hat{\gamma}_1(0) \ge 0$, the constants $l_{11}, l_{12}$ are positive adaptive gains, and $\sigma_i(t)$ satisfies $\sigma_i(t) > 0$, $\int_0^t \sigma_i(\tau)d\tau \le \bar{\sigma}_i$, $t \ge 0$, with some positive constant $\bar{\sigma}_i$, $i = 1, 2, 3$.

**Theorem 1.** *Consider the adaptive closed-loop system composed of the system (1) satisfying Assumptions 1 and 2, control law (4) and adaptive laws (5)–(6). Then, all closed-loop signals are bounded and* $\lim_{t\to\infty} E(t) = 0$.

**Proof.** From Ibrir et al. (2007), the dead-zone nonlinearity can be represented as $\Gamma(u(t)) = m(t)u(t) + h(t)$, where $m(t) = m_l$ or $m_r$, $|h(t)| \le \bar{h} = \max\{m_l, m_r\} \cdot \max\{b_l, b_r\}$. Define the Lyapunov–Krasovskii functional as

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t), \tag{7}$$

$$V_1(t) = \int_0^{W(t)} \rho(\xi)d\xi,$$

$$V_2(t) = \sum_{i=1}^n \int_{t-d_i(t)}^t \delta_{11}\alpha_i^2(2|E_i(\xi)|)d\xi,$$

$$V_3(t) = \frac{\eta}{2l_{11}}\tilde{\theta}_1^2(t), \qquad V_4(t) = \frac{\eta}{2l_{12}}\tilde{\gamma}_1^2(t),$$

where $\eta = \min\{m_l, m_r\}$, $\delta_{11} > 0$ is a constant, $\tilde{\theta}_1(t) = \theta_1^* - \hat{\theta}_1(t)$ and $\tilde{\gamma}_1(t) = \gamma^* - \hat{\gamma}_1(t)$ are the parameter estimation errors. From (3), it follows that

$$\begin{aligned} \dot{E}(t) &= (A + BK)E(t) + B[f - KE(t) \\ &\quad + m(t)u(t) + h(t) - y_d^{(n)}(t)], \\ \dot{W} &= -E^T QE + 2E^T PB[f - KE + m(t)u + h - y_d^{(n)}], \\ \dot{V}_1 &= -\rho(W)E^T QE + 2\rho(W)E^T PBf \\ &\quad - 2\rho(W)E^T PBKE + 2\rho(W)E^T PB(h - y_d^{(n)}) \\ &\quad + 2\rho(W)E^T PBm(t)u. \end{aligned} \tag{8}$$

By noting Assumption 1 and applying Young's inequality, it is shown that

$$\begin{aligned} 2\rho(W)E^T PBf &\le 2\rho(W)|E^T PB|\sum_{i=1}^n \theta_i\alpha_i(2|E_i(t - d_i(t))|) \\ &\quad + 2\rho(W)|E^T PB|\left[\sum_{i=1}^n \theta_i\alpha_i(2\bar{y}_{d_{i-1}}) + \gamma\right] \\ &\le \sum_{i=1}^n \delta_{11}(1 - \bar{d}_i)\alpha_i^2(2|E_i(t - d_i(t))|) \\ &\quad + \sum_{i=1}^n \frac{\theta_i^2}{\delta_{11}(1 - \bar{d}_i)}\rho^2(W)(E^T PB)^2 \\ &\quad + 2\rho(W)|E^T PB|\left[\sum_{i=1}^n \theta_i\alpha_i(2\bar{y}_{d_{i-1}}) + \gamma\right], \\ &\quad - 2\rho(W)E^T PBKE \le \delta_{12}lE^T E + \delta_{12}^{-1}\rho^2(W)(E^T PB)^2, \\ 2\rho(W)E^T PB(h - y_d^{(n)}) &\le 2\rho(W)|E^T PB|(\bar{h} + \bar{y}_{d_n}), \end{aligned} \tag{9}$$

where $\bar{y}_{d_{i-1}} = \sup_{t\ge 0}|y_d^{(i-1)}(t)|$, $i = 1, 2, \ldots, n + 1$, $\delta_{12} > 0$, $l > 0$, are constants, and $l$ is chosen such that $K^T K \le lI$. Substituting the inequalities (9) into (8), we have

$$\begin{aligned} \dot{V}_1 &\le -\rho(W)E^T QE + \delta_{12}lE^T E + \sum_{i=1}^n \delta_{11}(1 - \bar{d}_i) \\ &\quad \cdot \alpha_i^2(2|E_i(t - d_i(t))|) + \eta\theta_1^*\rho^2(W)(E^T PB)^2 \\ &\quad + \eta\gamma^*\rho(W)|E^T PB| + 2\rho(W)E^T PBm(t)u, \end{aligned} \tag{10}$$