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Brief paper

Event-triggered tracking control of unicycle mobile robots☆

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ABSTRACT

We investigate the stabilization of time-varying trajectories for unicycle mobile robots using event-triggered controllers. We follow an emulation-like approach in the sense that we first synthesize the controller while ignoring the communication constraints and we then derive an appropriate triggering condition. The solutions to the robot model are proved to practically converge towards the given reference trajectory, under some condition on the latter. Furthermore, the existence of a uniform minimum amount of time between any two transmissions is ensured. Afterward, experimental results are presented where the controller has been implemented on a remote computer which transmits its output to the mobile robot via an IEEE 802.11g wireless network. The proposed event-triggering strategy is able to significantly reduce the need for communication compared to a classical time-triggered setup while ensuring similar, if not better, tracking performances.

1. Introduction

The objective of the paper is to guarantee the tracking of given reference trajectories by unicycle mobile robots using a remote controller, while reducing the usage of the communication channel. In particular, we want to limit the amount of control input updates to save communication resources and to reduce the risk of packet losses and long transmission delays; noting that also allows to curb the actuators wear and to reduce the energy consumption of the actuators. An event-triggered feedback law is designed for that purpose. The idea of event-triggered control is to transmit data between the controller and the plant whenever a state-dependent criterion is satisfied and not periodically as in traditional setups. In that way, the transmissions are adapted to the state of the system and these only occur when it is needed. Various techniques have been developed, see Arzén (1999), Aström and Bernhardsson (2002), Heemels, Sandee, and van den Bosch (2009), Postoyan, Tabuada, Nešić, and Anta (2015) and Tabuada (2007) to mention a few. Most of them address the stabilization of equilibrium points, while very few controllers have been synthesized to stabilize time-varying trajectories, see *e.g.*, Tallapragada and Chopra (2013). It appears that tracking control induces additional difficulties as only *approximate* tracking can usually be ensured under communication constraints because of the time-varying component of the control law, see for more details (Postoyan, van de Wouw, Nešić, & Heemels, 2014). As a consequence, available results on the stabilization of equilibrium points are not directly applicable in this context.

We follow an emulation-like approach (see *e.g.*, Postoyan et al., 2015, Tabuada, 2007). Thus, we first design the controller while ignoring the communication constraints and we derive the triggering strategy afterward. We have selected the state-feedback controller of Jiang and Nijmeijer (1997) among others, because the

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law is continuous and an explicit Lyapunov function is provided. The continuity of the feedback law is useful to guarantee the existence of a minimum amount of time between two transmissions, which is essential in practice as the hardware cannot transmit infinitely fast. The Lyapunov function is used to design the event-triggering condition. This Lyapunov function is *weak* in the sense that it does not strictly decrease along the solutions of the closed-loop system in the absence of communication constraints, see Jiang and Nijmeijer (1997). To overcome this issue, we impose conditions on the reference trajectories under which the Lyapunov function strictly decreases outside a given neighborhood of the origin, which can be made as small as desired by appropriately tuning the control law parameters. We then take into account the effects of the network and we model the problem as a hybrid system using the formalism of Goebel, Sanfelice, and Teel (2012) (as in *e.g.*, Donkers & Heemels, 2012, Postoyan et al., 2015, Seuret, Prieur, & Marchand, 2013). The event-triggering condition we construct is inspired by the technique in Forni, Galeani, Nešić, and Zaccarian (2014) and Tabuada (2007) where the Lyapunov function is forced to decrease at a certain rate, except in the aforementioned neighborhood of the origin, which we call a *dead-zone*. We prove that the solutions of the robot model practically converge towards the reference trajectories. The ultimate bound depends on a tuneable parameter which corresponds to the 'size' of the dead-zone of the triggering condition. It can be used to adjust the accuracy of the tracking error at the price of more transmissions. The approach we follow is similar to the one in Tallapragada and Chopra (2013), however we do not impose the same conditions on the reference trajectories and we propose a different triggering law. Finally, we have implemented the event-triggered controller on a benchmark. The controller sends its data to the robot over a wireless network according to the triggering condition and the measurements are periodically collected by cameras which are directly connected to the remote controller via a dedicated high bandwidth wired channel. The experimental results show that it is possible to significantly reduce the usage of the wireless network compared to periodic sampling, if we agree to slightly give up on the tracking accuracy.

A similar problem is investigated in Santos, Mazo, and Espinosa (2012) where self-triggered controllers are developed. Besides the fact that we consider a nonlinear model of the robot and not a linear model as in Santos et al. (2012), we also envision a different setup. In our case, no local controller is implemented on the robot which helps saving batteries and the measurements are given by cameras which are connected via a wired channel to the controller unit (as opposed to odometry in Santos et al. (2012)), which justifies the choice of an event-triggered implementation. Compared to our preliminary work in Postoyan et al. (2013), the results rely on a simplified stability analysis and on different assumptions on the reference trajectories which cover a class of well-motivated cases. In particular, we are able to guarantee the existence of a minimum amount of time between two transmissions for both of the reference trajectories we consider in the experimental part, which is not the case of Postoyan et al. (2013). Furthermore, we present new experimental results as we use cameras to measure the robot position as explained above and not odometry as in Postoyan et al. (2013); noting that the latter has the drawback to use the wireless network periodically to communicate with the controller.

The paper is organized as follows. Preliminaries are given in Section 2. In Section 3, the controller of Jiang and Nijmeijer (1997) is recalled and the hybrid model is introduced. The event-triggering condition and the analytical results are presented in Section 4. Section 5 deals with the experimental results. The proofs are provided in the Appendix.

2. Preliminaries

Let $\mathbb{R} = (-\infty, \infty)$, $\mathbb{R}_{\geq 0} = [0, \infty)$, $\mathbb{R}_{>0} = (0, \infty)$, $\mathbb{Z}_{\geq 0} = \{0, 1, 2, \ldots\}$, and $\mathbb{Z}_{>0} = \{1, 2, \ldots\}$. For $(x, y) \in \mathbb{R}^{n+m}$, the notation (x, y) stands for $[x^{\mathrm{T}}, y^{\mathrm{T}}]^{\mathrm{T}}$. A function $\gamma : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is of class $\mathcal{K}$ if it is continuous, zero at zero and strictly increasing, and it is of class $\mathcal{K}_\infty$ if in addition it is unbounded. A continuous function $\gamma : \mathbb{R}^2_{\geq 0} \longrightarrow \mathbb{R}_{\geq 0}$ is of class $\mathcal{KL}$ if for each $t \in \mathbb{R}_{\geq 0}$, $\gamma(\cdot, t)$ is of class $\mathcal{K}$, and, for each $s \in \mathbb{R}_{>0}$, $\gamma(s, \cdot)$ is decreasing to zero. For a right-continuous function $f : \mathbb{R} \to \mathbb{R}^n$, $f(t^+)$ stands for $\lim_{s\to t,\, s>t} f(s)$ for $t \in \mathbb{R}$. We recall that the function sinc is defined from $\mathbb{R}$ to $\mathbb{R}$ by $\mathrm{sinc}(x) = \frac{\sin x}{x}$ when $x \neq 0$, and $\mathrm{sinc}(0) = 1$, and it is twice continuously differentiable with $\mathrm{sinc}'(0) = 0$ and $\mathrm{sinc}''(0) = -\frac{1}{3}$.

We will write the event-triggered controlled system as a hybrid system using the formalism of Goebel et al. (2012). In particular, we will consider a system of the form

$$\dot{x} = f(x) \quad x \in C, \qquad x^+ = g(x) \quad x \in D, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state and $C, D \subset \mathbb{R}^n$ are respectively the flow and the jump sets. We assume that f and g are continuous and that C and D are closed sets (this will be the case in the paper). We recall some basic definitions, see Goebel et al. (2012). A set $E \subset \mathbb{R}_{\geq 0} \times \mathbb{Z}_{\geq 0}$ is a *compact hybrid time domain* if $E = \cup_{j\in\{0,\ldots,J-1\}}([t_j, t_{j+1}], j)$ for some finite sequence of times $0 = t_0 \leq t_1 \leq \cdots \leq t_J$. The set E is a *hybrid time domain* if for all $(T, J) \in E$, $E \cap ([0, T] \times \{0, 1, \ldots, J\})$ is a compact hybrid time domain. A *hybrid arc* is a function ϕ defined on a hybrid time domain $\mathrm{dom}\,\phi$ and such that, for each $j \in \mathbb{Z}_{\geq 0}$, $t \mapsto \phi(t, j)$ is locally absolutely continuous on $I^j := \{t : (t, j) \in \mathrm{dom}\,\phi\}$. A hybrid arc $\phi : \mathrm{dom}\,\phi \to \mathbb{R}^n$ is a *solution* to (1) if: (i) $\phi(0, 0) \in C \cup D$; (ii) for all $j \in \mathbb{Z}_{\geq 0}$ and almost all $t \in I^j$, $\phi(t, j) \in C$ and $\dot{\phi}(t, j) = f(\phi(t, j))$; (iii) for $(t, j) \in \mathrm{dom}\,\phi$ such that $(t, j + 1) \in \mathrm{dom}\,\phi$, $\phi(t, j) \in D$ and $\phi(t, j + 1) = g(\phi(t, j))$. A solution to (1) is said to be: *nontrivial* if $\mathrm{dom}\,\phi$ contains at least two points; *maximal* if it cannot be extended; *complete* if $\mathrm{dom}\,\phi$ is unbounded.

The definition below characterizes hybrid systems that generate solutions for which two successive jumps are spaced by (at least) a strictly positive amount of time which is uniform over the ball of initial conditions (see Postoyan et al., 2015).

Definition 1. The solutions to (1) have *a uniform semiglobal dwell-time* if for any $\Delta \geq 0$, there exists $\eta(\Delta) > 0$ such that for any solution ϕ to (1) with $|\phi(0, 0)| \leq \Delta$,

$$\sup I^j - \inf I^j \geq \eta(\Delta) \quad \forall j \in \mathbb{Z}_{>0} \text{ with } I^j \neq \emptyset. \quad \square \tag{2}$$

We recall the following invariance definition (see Goebel et al., 2012).

Definition 2. Consider system (1), the set $\mathcal{A} \subset \mathbb{R}^n$ is **strongly pre-forward invariant** if for any $\xi \in \mathcal{A}$ each solution ϕ with initial condition ξ satisfies $\phi(t, j) \in \mathcal{A}$ for all $(t, j) \in \mathrm{dom}\,\phi$. $\square$

3. System model

3.1. Jiang and Nijmeijer's controller

We consider a mobile robot for which the dynamics are defined by the following nonholonomic system:

$$\dot{x} = v\cos(\theta), \qquad \dot{y} = v\sin(\theta), \qquad \dot{\theta} = w, \tag{3}$$

where (x, y) are the Cartesian coordinates and θ is the angle between the heading direction and the x-axis, (v, w) denotes the control input vector, see Fig. 1. The objective is to make the solution to