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Brief paper

On input allocation-based regulation for linear over-actuated systems[☆]

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ABSTRACT

Results concerning the output regulation problem for over-actuated linear systems are presented in this paper. The focus is on the characterization of the solution of the full-information regulator problem for systems which are right-invertible (but not left-invertible) and the input operator is injective. The intrinsic redundancy in the plant model is exploited by parameterizing all solutions of the regulator equations and performing a static or dynamic optimization on the space of solutions. This approach effectively shapes the non-unique steady-state of the system so that the long-term behavior optimizes a given performance index. In particular, nonlinear cost functions that account for constraints on the inputs are considered, within the general form of a hybrid system assumed for the allocation mechanism. An example is given to illustrate the proposed methodology.

1. Introduction

Traditionally, the presence of a redundant set of control inputs in a given control system (defined as the availability of a larger number of control inputs than regulated outputs) is addressed either by "squaring down" the plant model (Saberi & Sannuti, 1988) or by resorting to *control allocation* (Bodson, 2002; Harkegård & Glad, 2005; Johansen & Fossen, 2013). In particular, many variations on the theme of this latter methodology – quite popular in vehicular applications, noticeably flight control – assume that a virtual control input can be defined, which has the same dimension as the regulated output (see Johansen & Fossen, 2013 and references therein). The control strategy designed on the basis of this virtual input is then "distributed" across the redundant set of actuators via optimization of a given cost function. Notwithstanding the fact that multiple actuators are often necessary for technological reasons, an optimal design of the allocation stage has shown to lead to strong advantages in a broad range of applications (ranging from the aerospace to the automotive and several other industrial fields), both in terms of saturation handling (De Tommasi, Galeani, Pironti, Varano, & Zaccarian, 2011; Zaccarian, 2009) and in terms of fulfillment of more general performance goals (Boncagni et al., 2012; Cordiner, Galeani, Mecocci, Mulone, & Zaccarian, 2014; Passenbrunner, Sassano, & Zaccarian, 2012; Trégouët, Arzelier, Peaucelle, Pittet, & Zaccarian, in press; Zhou, Fiorentini, Canova, & Serrani, 2013).

For systems that are affine in the control, it is typically assumed that the input redundancy lies completely in the null-space of the input matrix. Clearly, this scenario does cover all the possibilities, as injective input operators can still be considered. Input redundancy with full-rank input operators has been termed *weak input redundancy* in Zaccarian (2009), where a taxonomy of over-actuated linear systems was proposed on the basis of the distinction between the null space of the control input matrix versus the null space of the multivariable DC gain of the plant model. In a state-space setting, weakly input redundant systems are characterized by multiple independently controllable state trajectories

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that are compatible with a given output. Specifically, the trajectories of the inverse model are not uniquely determined by the initial conditions, hence the possibility exists to modify redundant steady-state motions that are all compatible with a given output reference. This feature is exploited in this paper within the framework of (full-information) output regulation theory.

To the best of our knowledge, the *output regulation problem* for linear over-actuated systems has been investigated first in Sigthorsson and Serrani (2006) in the context of tracking control for a linearized model of a hypersonic aircraft, and later extended to encompass linear parameter-varying models within the considered application (Sigthorsson, Serrani, Bolender, & Doman, 2009). The steady-state optimization for an input-redundant linear system with nonlinear output function has been considered in Johansen and Sbarbaro (2005), with exosystem model restricted to pure integrators. For the same type of exosystem, the results in De Tommasi et al. (2011) and Zaccarian (2009) provide a framework allowing for nonlinear dynamic allocation solutions. This very framework has been in turn adopted in Serrani (2012), where the output regulation problem for strictly proper over-actuated LTI models is approached by resorting to a *redundant servo-mechanism* that directly allocates the trajectories of the plant inverse model. A different approach, aimed at achieving output regulation by exploiting *nonlinear solutions* of the *linear* Francis equations, has been considered in Galeani and Valmorbida (2013) for exosystems restricted to pure integrators, and in Valmorbida and Galeani (2013) for Poisson stable exosystems.

In this paper, we consider the full-information output regulation problem for non-strictly proper, over-actuated LTI models by restricting our attention to the weakly input-redundant case. The focus of the paper is on the characterization in geometric terms (and the ensuing parameterization) of the redundancy provided by an infinite number of solutions to the regulator equations. This parameterization is then exploited by an appropriate allocation mechanism, which in its most general form takes the structure of a hybrid system. The geometric properties of the solution of the regulator equations are then invoked to determine the most suitable structure of the compensator on the basis of the specific allocation policy adopted (i.e., constant or time-varying) and the corresponding behavior of the resulting reference motion.

The paper is organized as follows: background material is presented in Section 2, where the problem is formally stated. In Section 3 the properties of the solution of the regulator equation are discussed, and the structure of the allocation model is proposed. The selection of the allocation policy is discussed in Section 4. Finally, an illustrative simulation example is presented and discussed in Section 5, and conclusions are offered in Section 6.

Notation: For a matrix S, $\ker S$ denotes its kernel, $\operatorname{im} S$ denotes its image, and if S is square, spec (S) denotes its spectrum (the set of its eigenvalues). $\mathbb{C}^0$ denotes the set of purely imaginary complex numbers.

2. Background and problem statement

We consider linear systems of the form

$$\dot{w} = Sw \tag{1a}$$

$$\dot{x} = Ax + Bu + Pw \tag{1b}$$

$$e = Cx + Du + Qw \tag{1c}$$

with state $w \in \mathbb{R}^q$ and $x \in \mathbb{R}^n$, control input $u \in \mathbb{R}^m$ and performance output $e \in \mathbb{R}^p$. Following standard terminology in output regulation theory (Davison & Goldenberg, 1975; Francis, 1977), $\mathcal{P} := (A, B, C, D)$ is referred to as the realization of the *plant* and $\mathcal{S} := (S, P, Q)$ as the realization of the *exosystem*. The following assumptions define the class of models considered in this paper.

Assumption 1. (1) $\mathcal{P}$ is *over-actuated*, $m > p$;
(2) $\operatorname{rank} B = m$ and $\operatorname{rank} C = p$;
(3) (A, B) is stabilizable;
(4) The matrix S is semi-simple (that is, it has only simple eigenvalues) and $\operatorname{spec}(S) \subset \mathbb{C}^0$.

Remark 1. Item 2 of Assumption 1 is required to avoid trivialities and overlap with previous results in Zaccarian (2009). The case of a rank-deficient B, which corresponds to having the so-called *strong input redundancy* for (A, B), can be handled separately from the weak input redundancy exploited here, essentially by projection modulo $\ker B$.⌟

The problem addressed in this paper is the design of a *full information* (possibly nonlinear) *regulator* that is capable of exploiting the redundancy stated at item 1 of Assumption 1 to induce a *desirable selection* of the control input u (in a sense to be specified). As customary, by full information it is meant that both x and w are available for measurement. Here, it is also assumed that $\mathcal{P}$ and $\mathcal{S}$ are known exactly. As for the possibilities offered by over-actuation, we consider in Section 4 the minimization of a functional that corresponds to keeping the steady state input away from the saturation limits. As pointed out in Zaccarian (2009), the use of input allocation in the presence of input saturation should be seen as synergistic with anti-windup techniques, since the latter account for saturation during transients, whereas the former addresses steady-state saturations. This must be accomplished while guaranteeing asymptotic stability of the controllable modes of (1) and the asymptotic tracking requirement $\lim_{t\to\infty} e(t) = 0$.

Define the *system matrix* of $\mathcal{P}$ as $P_\Sigma(s) := \begin{bmatrix} A - sI & B \\ C & D \end{bmatrix}$ (see Rosenbrock, 1970). Under Assumption 1, a well-known sufficient condition for the solvability of the regulator problem, which is necessary if *generic solvability* is considered (i.e., for all matrices $P \in \mathbb{R}^{n\times q}$ and $Q \in \mathbb{R}^{p\times q}$), is given by:

Assumption 2. The set of invariant zeros of $\mathcal{P}$ is disjoint from the spectrum of S, that is, $\operatorname{rank} P_\Sigma(\lambda) = n + p,\ \forall\, \lambda \in \operatorname{spec} S$.

Assumption 2 implies that $\mathcal{P}$ is *non-degenerate* (Hewer & Martin, 1984); this, together with Assumption 1.1, implies that $\mathcal{P}$ is *right-invertible*. Recall that $\mathcal{P}$ is left (right) invertible if and only if $\operatorname{rank} P_\Sigma(s) = n + m$ ($\operatorname{rank} P_\Sigma(s) = n + p$) as a polynomial matrix; obviously, an over-actuated system is not left-invertible. *Left invertibility* is equivalent to the fact that the applied input can be uniquely recovered from the forced response output, whereas *right invertibility* implies that any sufficiently smooth function can be reproduced as a forced output response.

Finally, we recall a few geometric concepts that will be used in the sequel. By $\mathcal{V}^* \subset \mathbb{R}^n$, we denote the *weakly unobservable subspace* for $\mathcal{P}$, i.e., the set of initial conditions for which there exists an input function such that the ensuing output is identically zero. It is well known (Trentelman, Stoorvogel, & Hautus, 2001) that $\mathcal{V}^*$ is the largest subspace $\mathcal{V} \subset \mathbb{R}^n$ such that

$$\begin{bmatrix} A \\ C \end{bmatrix} \mathcal{V} \subset (\mathcal{V} \times 0) + \operatorname{im} \begin{bmatrix} B \\ D \end{bmatrix}, \tag{2}$$

or equivalently the largest subspace $\mathcal{V} \subset \mathbb{R}^n$ such that there exists $F \in \mathbb{R}^{m\times n}$ ensuring

$$(A + BF)\mathcal{V} \subset \mathcal{V}, \qquad (C + DF)\mathcal{V} = 0. \tag{3}$$

A matrix F satisfying (3) is called a *friend of* $\mathcal{V}$. Similarly, we denote by $\mathcal{R}^* \subset \mathbb{R}^n$ the *controllable weakly unobservable subspace*[1] of $\mathcal{P}$,

[1] When $D = 0$, $\mathcal{V}^*$ and $\mathcal{R}^*$ are usually termed respectively the *largest controlled-invariant subspace* and the *largest controllability subspace* contained in $\ker C$ (Wonham, 1985).