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Brief paper

Finite-horizon reliable control with randomly occurring uncertainties and nonlinearities subject to output quantization☆

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ABSTRACT

This paper deals with the finite-horizon reliable $\mathcal{H}_\infty$ output feedback control problem for a class of discrete time-varying systems with randomly occurring uncertainties (ROUs), randomly occurring nonlinearities (RONs) as well as measurement quantizations. Both the deterministic actuator failures and probabilistic sensor failures are considered in order to reflect the reality. The actuator failure is quantified by a deterministic variable varying in a given interval and the sensor failure is governed by an individual random variable taking value on $\left[0, 1\right]$. Both the nonlinearities and the uncertainties enter into the system in random ways according to Bernoulli distributed white sequences with known conditional probabilities. The main purpose of the problem addressed is to design a time-varying output feedback controller over a given finite horizon such that, in the simultaneous presence of ROUs, RONs, actuator and sensor failures as well as measurement quantizations, the closed-loop system achieves a prescribed performance level in terms of the $\mathcal{H}_\infty$-norm. Sufficient conditions are first established for the robust $\mathcal{H}_\infty$ performance through intensive stochastic analysis, and then a recursive linear matrix inequality approach is employed to design the desired output feedback controller achieving the prescribed $\mathcal{H}_\infty$ disturbance rejection level. A numerical example is given to demonstrate the effectiveness of the proposed design scheme.

1. Introduction

For several decades, stochastic control and nonlinear control serve as two of the most active research areas in systems and control that have found successful applications in a variety of engineering systems such as automotive engines, robot manipulators, aircraft and electrical motors. So far, considerable research attention has been devoted to the theoretical research on control problems for nonlinear stochastic systems, see Basin and Rodriguez-Ramirez (2014), Caballero-Aguila, Hermoso-Carazo, and Linares-Perez (2014), Ding, Wang, Hu, and Shu (2013), Ding, Zhang, Yin, and Ding (2013), Hernandez-Gonzalez and Basin (2014), Karimi, Maralani, and Moshiri (2006), Shen, Wang, and Liu (2012), Tong, Li, and Wang (2013), Wang, Ho, Liu, and Liu (2009), Yin, Ding, and Luo (2013) and the references therein. For example, for different kinds of nonlinear stochastic systems, the $\mathcal{H}_\infty$ output feedback control problem has been investigated in Wang et al. (2009), the adaptive fuzzy control problem has been proposed in Tong et al. (2013), the neural-network-based controller design has been addressed in Wang, Tong, and Li (2013), the adaptive sliding mode controller has been designed in Chen, Niu, and Zou (2013), Hu, Wang, Gao, and Stergioulas (2012), Niu, Ho, and Wang (2008) and Niu, Liu, and Jia (2013) and the observer-based control problems have been solved in Wang, Lam, Ma, Bo, and Guo (2011), respectively. Among various descriptions of nonlinearities, the so-called randomly occurring nonlinearities (RONs) (Wang, Wang, & Liu, 2010) cater for those randomly changeable nonlinearities in terms of their types and/or intensities governed by stochastic variables. RONs, which typically occur in networked environments, encompass several well-studied nonlinearities in stochastic systems and have thus stirred particular research interests in the past few years.

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It is noticeable that, in almost all the aforementioned literature, the components of the control systems have been implicitly assumed to be fully reliable. This assumption is, unfortunately, not always true since the failures of control components (e.g. sensors and actuators) often occur in practical applications due to a variety of reasons, for example, the abrupt changes of working conditions, the erosion caused by severe circumstance, the internal component constraints, the intense external disturbance and the aging of sensors or actuators, etc. Therefore, it is of both practical significance and theoretical importance to design a reliable controller against possible actuator and sensor failures such that the essential performance of the controlled system can be guaranteed (Bouibed, Seddiki, Guelton, & Akdag, 2014; Liu, Wang, He, & Zhou, 2014; Qin, He, & Zhou, 2014). In fact, in the past two decades, the problem of reliable controller design has attracted much research attention and many approaches have been proposed in the literature including Hamilton–Jacobi equation approach (Liang & Xu, 2006; Liu, Wang, & Yang, 1998), robust pole region assignment technique (Gundes & Ozbay, 2010), algebraic Riccati equation approach (Suchomski, 2007; Yang, Wang, & Soh, 2001) and linear matrix inequality approach (Jiang, Yang, & Shi, 2010; Ma & Yang, 2009; Wong, Tian, Yue, & Au, 2013). Despite the fruitful results on *time-invariant systems* over an *infinite horizon*, it is worth pointing out that the *finite-horizon* reliable control problem for *time-varying* systems has not been thoroughly investigated yet, not to mention the case complicated further by nonlinearity and stochasticity.

As is well known, modeling errors (usually parameter uncertainties) constitute an important kind of complexities for system modeling that has a great impact on the subsequent system analysis and design (Wu, Zheng, & Gao, 2013). In today's networked systems, it is quite common that the network load is randomly fluctuated and the signal transmission suffers from unpredictable networked-induced phenomena owing to limited bandwidth. As such, the occurrence of the parameter uncertainties in a networked environment is often of random nature resulting from abrupt phenomena such as modification of the operating point of a linearized model of nonlinear systems, random failures and repairs of the components, changes in the interconnections of subsystems and sudden environment disturbances, etc. Very recently, in Hu et al. (2012), the concept of randomly occurring uncertainties (ROUs) has been introduced to reflect the probabilistic fashion of the network-induced modeling error and the corresponding sliding mode control problem has been considered. However, when it comes to the finite-horizon reliable $\mathcal{H}_\infty$ control problem involving ROUs for nonlinear time-varying stochastic systems, the related results are very few and the situation is even worse when measurement quantizations, actuator and sensor failures are also taken into account. It is, therefore, the aim of this paper to shorten such a gap by addressing the finite-horizon reliable $\mathcal{H}_\infty$ control problem with presence of ROUs, RONs as well as measurement quantizations.

Motivated by the above discussion, in this paper, we launch a major study on the finite-horizon reliable $\mathcal{H}_\infty$ output-feedback control problem for a class of discrete time-varying stochastic systems with simultaneous presence of actuator and sensor failures, ROUs, RONs and measurement quantizations. Some sufficient conditions are established, via intensive stochastic analysis, to guarantee the existence of the desired time-varying output feedback controller gains, and then such controller gains are characterized by solving a set of recursive matrix inequalities. A simulation example is finally presented to illustrate the effectiveness of the proposed design scheme. The main contributions of this paper are highlighted as follows. (1) The problem addressed is new in the sense that this paper represents the first of few attempts to deal with the finite-horizon reliable $\mathcal{H}_\infty$ output feedback control problem against actuator and sensor failures for a class of discrete time-varying stochastic systems. (2) The system under consideration is comprehensive to cover time-varying parameters, actuator and sensor failures, ROUs, RONs and measurement quantizations, hence reflecting the reality more closely. (3) The algorithm developed is computationally appealing in terms of its recursive nature suitable for online application.

The rest of this paper is outlined as follows. In Section 2, a class of discrete time-varying stochastic systems is presented with actuator and sensor failures, ROUs, RONs and measurement quantizations. The problem under consideration is formulated. In Section 3, by employing the stochastic analysis techniques, some sufficient conditions are established to guarantee the desired output feedback controller performances and then the controller gains are obtained by solving a set of recursive matrix inequalities. A simulation example is given in Section 4 to demonstrate the main results obtained. Finally, we conclude the paper in Section 5.

Notation. The notation used here is standard except where otherwise stated. $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n \times m$ real matrices. $l[0, N]$ is the space of vector functions over $[0, N]$. The notation $X \geq Y$ (respectively, $X > Y$), where X and Y are real symmetric matrices, means that $X - Y$ is positive semi-definite (respectively, positive definite). M^T represents the transpose of the matrix M. 0 represents zero matrix of compatible dimensions. The n-dimensional identity matrix is denoted as I_n or simply I, if no confusion is caused. diag$\{\cdots\}$ stands for a block-diagonal matrix. $\mathbb{E}\{x\}$ and $\mathbb{E}\{x|\,y\}$ will, respectively, denote expectation of the stochastic variable x and expectation of x conditional on y. Prob$\{\cdot\}$ means the occurrence probability of the event "$\cdot$". In symmetric block matrices, "$*$" is used as an ellipsis for terms induced by symmetry. The symbol $\otimes$ denotes the Kronecker product. Matrices, if they are not explicitly specified, are assumed to have compatible dimensions.

2. Problem formulation

Consider the following uncertain discrete time-varying nonlinear stochastic system defined on $k \in [0, N]$:

$$\begin{cases} x(k+1) = (A(k) + \alpha(k)\Delta A(k))x(k) + B_1(k)u(k) \\ \qquad + \beta(k)g(k, x(k)) + D(k)w(k) \\ z(k) = M(k)x(k) + B_2(k)u(k) \\ x(0) = \varphi_0 \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ represents the state vector; $u(k) \in \mathbb{R}^{n_u}$ is the control input; $z(k) \in \mathbb{R}^{n_z}$ is the controlled output; $w(k) \in \mathbb{R}^{n_w}$ is the disturbance input which belongs to $l[0, N]$; and φ_0 is a given real initial value. $A(k)$, $B_1(k)$, $B_2(k)$, $D(k)$ and $M(k)$ are known, real, time-varying matrices with appropriate dimensions.

The nonlinear function $g(k, x(k))$ satisfies the following condition:

$$\|g(k, x(k))\|^2 \leq \varepsilon(k)\|E(k)x(k)\|^2 \tag{2}$$

where $\varepsilon(k) > 0$ is a known positive scalar and $E(k)$ is a known time-varying matrix.

The real-valued matrix $\Delta A(k)$ represents the norm-bounded parameter uncertainties of the following structure

$$\Delta A(k) = H_a(k)F(k)N(k), \tag{3}$$

where $H_a(k)$ and $N(k)$ are known time-varying matrices, while $F(k)$ is an unknown time-varying matrix function satisfying the following condition

$$F^T(k)F(k) \leq I. \tag{4}$$