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A generalized reaching law for discrete time sliding mode control☆

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ABSTRACT

The paper presents a generalized algorithm for the reaching and boundedness of the sliding variable in case of discrete time sliding mode control systems. The algorithm is general as it not only comprises functions in the sliding variable, but also functions in other variables, which are known. With the help of the proposed generalized reaching law, an important problem of a reaching law with uncertainty bounded by known functions is studied. The convergence of the sliding variable to an ultimate bounding function in presence of such an uncertainty is shown by the band approach method. This is achieved by appropriately selecting the functions in the proposed generalized reaching law.

1. Introduction

Since the advent of digital computation, most continuous systems are treated in their discretized forms. Hence, a major focus is on the discrete sliding mode, which has been dealt in various literature (Bandyopadhyay & Janardhanan, 2006; Bartoszewicz, 1998; Bertolini, Ferrara, & Utkin, 1995; Gao, Wang, & Homaifa, 1995). All these works mainly deal with proposing a reaching law and deriving the control and the ultimate band in terms of the controller parameters used in the reaching law. The most popular among these reaching laws are the ones attributed to Gao (Gao et al., 1995) and Utkin (Bertolini et al., 1995).

In the recent work (Chakrabarty & Bandyopadhyay, 2012), a new approach to analyze Gao's discrete reaching law has been developed and utilized to find out the controller parameter values once the desired ultimate band value is chosen from a specified range. This approach has been proved to be better than Gao's approach in Gao et al. (1995), since the ultimate band obtained this way is much lesser than that proposed by Gao.

1.1. Motivation

This new approach in Chakrabarty and Bandyopadhyay (2012), which the authors refer to as the band approach method, has not been used to analyze any dynamics other than Gao's reaching law until recently in Chakrabarty and Bandyopadhyay (2013), where the sliding variable dynamics as generated by the system is found out to be different from the reaching law used to derive the control. The dynamics was not only a function of the sliding variable but also had the states of the system mixed in it. The band approach had been applied to analyze this dynamics and the ultimate band was found out and controller parameters were computed. This work had put forward a more complicated dynamics than Gao's reaching law, which was subjected to the band approach method. This inspired the authors to work farther and search for a more generalized dynamics that this analysis method can handle. The work in this paper is a result of that investigation as of now.

1.2. Contribution and scope

With the help of the different functions in the generalized reaching law, one may carefully design the dynamics of the sliding variable. These designs can vary from one problem to another. In the presented work, one such problem is discussed which studies the presence of an uncertainty bounded by known functions in the reaching algorithm. With appropriate choices of the functions and the parameters in the generalized reaching law, we can obtain an ultimate bounding function inside which the sliding variable converges. Such a problem of uncertainty bounded by known functions and convergence inside a bounding function rather than a fixed band had not been discussed in the earlier literature of discrete sliding mode control systems. The work presented in this paper is novel in that regard, and could have been achieved by the proposed generalized reaching law.

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One needs to mention at this point that there had been several works in the literature which deal with proposing new reaching laws or modifying the already popular reaching laws (Niu, Ho, & Wang, 2010; Yuan, Shen, Xiao, & Wang, 2012; Zhu, Wang, Jiang, & Wang, 2009). However, there has not been any proposal for any generalized reaching algorithm, which can be chosen for the purpose of studying various discrete sliding mode control problems. The work in this paper is a step towards this objective, and is novel in that regard.

Also, one needs to mention here that the generalized reaching law proposed is not a result of any explicit/implicit discretization of a continuous reaching law. However, due to the presence of $\text{sgn}(s_k)$ term instead of $\text{sgn}(s_{k+1})$, it has an explicit Euler discretization form. The analysis in this paper concerns only the convergence and ultimate boundedness of the sliding variable following the proposed discrete reaching law and does not concern any discretization analysis as presented in the works such as Acary and Brogliato (2010), and Galias and Yu (2006).

2. The generalized reaching algorithm

Let us propose a discrete time reaching law as

$$s_{k+1} = f_1(s_k) + f_2(\xi_k, k) + f_3\text{sgn}(s_k) + d_k \tag{1}$$

where f_1 and f_2 are functions in the variables mentioned, f_3 can either be a constant or a varying gain depending on a particular problem. We suppose $s_k \in \mathbb{R}$ and $\xi_k \in \mathbb{R}^q$ and k denoting the sample count. Here ξ_k can be any variable other than s_k, which is known at all k. The uncertainty d_k is assumed to vary only at the sampling instants and bounded.

3. Disturbance bounded by known functions

In this section, we show how the generalized reaching law (1), with appropriate selection of the functions f_1, f_2 and f_3 is able to deal with a disturbance bounded by known functions.

Let $f_1(s_k) = f_0 s_k$ in the reaching law (1). Rewriting the same below, we get

$$s_{k+1} = f_0 s_k + f_2(\xi_k, k) + f_3\text{sgn}(s_k) + d_k \tag{2}$$

where $d_k \in [d_{k-}, d_{k+}]$ assumed, d_{k-} and d_{k+} are known and bounded functions such that $d_{k-} < d_{k+}\ \forall\ k$. This means that d_k is any value with mean $d_{k0} = (d_{k+} + d_{k-})/2$ and spread $d_{k1} = (d_{k+} - d_{k-})/2$. Obviously, d_{k0} and d_{k1} are also known but varying functions.

3.1. Main analysis

Lemma 1. *In the region $|s_k| > B_d$ with $B_d \in [d_{k1}, 2d_{k1}]$ (i.e., B_d is a known but varying positive function), $|s_{k+1}| < |s_k|$ is ensured for the algorithm (2) if $f_3 = (1 - f_0)B_d - d_{k1}$, assuming $f_2(\xi_k, k) = -d_{k0}$.*

Proof. In the region $s_k > B_d$, $|s_{k+1}| < |s_k| (\Rightarrow s_{k+1} < s_k)$ for the algorithm (2) is ensured when

$$s_k > \frac{f_2 + f_3 + d_{k+}}{1 - f_0}. \tag{3}$$

The above inequality is obtained by putting the condition $s_{k+1} < s_k$ in (2) and considering the maximum limit of the uncertainty $d_k = d_{k+}$ so as to incorporate every possibility.

Also, we suppose $s_{k+1} > -B_c$ for some $B_c \geq 0$ to avoid divergence while crossing $s_k = 0$. Using this in (2), the following condition is obtained.

$$s_k > \frac{-B_c - f_2 - f_3 - d_{k-}}{f_0} \tag{4}$$

considering the minimum limit of uncertainty $d_k = d_{k-}$ so as to incorporate every possibility.

Now, we make the RHS of the inequalities (3) and (4) equal to B_d, to ensure $s_k > B_d$ holds true always. Hence, we get

$$\frac{f_2 + f_3 + d_{k+}}{1 - f_0} = B_d \tag{5}$$

and

$$\frac{-B_c - f_2 - f_3 - d_{k-}}{f_0} = B_d. \tag{6}$$

Similarly, it can be shown that in the region $s_k < -B_d$, $|s_{k+1}| < |s_k|$ would be ensured again by (5) and (6).

Assuming $f_2 = -d_{k0}$, it is easy to see that $f_2 + d_{k+} = d_{k1}$ and $f_2 + d_{k-} = -d_{k1}$.

Hence, from (5), we get

$$f_3 = (1 - f_0)B_d - d_{k1}. \tag{7}$$

From (6) and using (7), we get

$$B_c = 2d_{k1} - B_d. \tag{8}$$

Now, since $B_c \geq 0$ and we can assume $B_c \leq B_d$ to avoid divergence, we get from (8)

$$d_{k1} \leq B_d \leq 2d_{k1}. \tag{9}$$

Lemma 2. *In the region $0 < s_k \leq B_d$, $-B_s \leq s_{k+1} \leq B_d$ is ensured for the algorithm (2) if $f_3 = (1 - f_0)B_d - d_{k1}$ and $f_0 \geq 0$, with $B_s = 2d_{k1} - (1 - f_0)B_d$. Similarly, in the region $0 > s_k \geq -B_d$, $B_s \geq s_{k+1} \geq -B_d$ is ensured for the algorithm (2) under the same conditions.*

Proof. In the region $0 < s_k \leq B_d$, using (7) from Lemma 1 in (2), we get the maximum limit for s_{k+1} as

$$s_{k+1} = f_0 B_d + f_2 + f_3 + d_{k+} = B_d \tag{10}$$

by taking the maximum limit of the disturbance and on the assumption $f_0 \geq 0$.

The minimum limit for s_{k+1} in this region can be obtained by taking $s_k \to 0$ and the minimum limit of the disturbance $d_k = d_{k-}$ in (2). This gives its value as

$$-B_s = f_2 + f_3 + d_{k-} \Longrightarrow B_s = 2d_{k1} - (1 - f_0)B_d. \tag{11}$$

Using a similar analysis, it can be shown that $s_{k+1} \leq B_s$ and $s_{k+1} \geq -B_d$ when $0 > s_k \geq -B_d$.

Theorem 1. *The absolute value of sliding variable s_k (i.e., $|s_k|$) in the discrete time algorithm (2) will be ultimately bounded inside $B_d \in [d_{k1}, 2d_{k1}]$ with*

(1) $f_0 = 2 - \frac{2d_{k1}}{B_d}$
(2) $f_3 = (1 - f_0)B_d - d_{k1}$.

Proof. From (11) of Lemma 2, it is not conclusive if $B_s < B_d$ or $B_s > B_d$. We shall have $|s_k|$ bounded by the greater of the two. To get this bound conclusively, let us equate B_d and B_s, such that

$$B_d = 2d_{k1} - (1 - f_0)B_d$$
$$\text{or,} \quad B_d = \frac{2d_{k1}}{2 - f_0}. \tag{12}$$

Now, B_d is a positive function bounded by d_{k1} and $2d_{k1}$ as per (9) of Lemma 1. With such a B_d chosen, one can calculate f_0 from (12) as

$$f_0 = 2 - \frac{2d_{k1}}{B_d} \tag{13}$$

and f_3 is already obtained in (7) of Lemma 1.