



Tensor modelling of MIMO communication systems with performance analysis and Kronecker receivers

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ABSTRACT

The purpose of this paper is manifold. In a first part, we present a new alternating least squares (ALS)-based method for estimating the matrix factors of a Kronecker product, the so-called Kronecker ALS (KALS) method. Four other methods are also briefly described. In a second part, we consider the design of multiple-input multiple-output (MIMO) wireless communication systems using tensor modelling. Eight systems are presented in a unified way, and their theoretical performance is compared in terms of maximal diversity gain. Exploiting a Kronecker product of symbol and channel matrices, and applying the algorithms introduced in the first part, we propose three semi-blind and two supervised receivers, called Kronecker receivers, for jointly estimating the channel and the transmitted symbols. Necessary identifiability conditions are established. Finally, extensive Monte Carlo simulation results are provided to compare the performance of three tensor-based systems, on the one hand, and of the five proposed Kronecker receivers for the tensor space-time-frequency (TSTF) coding system, on the other hand.

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1. Introduction

Kronecker products, also known as tensor products, of matrices are currently used in many signal and image processing applications, like in compressive sensing with Kronecker dictionaries [1] and for image restoration [2]. They are useful in system theory [3] and in numerical linear algebra to write and solve linear matrix equations like Lyapunov and more generally Sylvester equations [4]. They also play an important role to simplify and implement fast transform algorithms like fast Fourier, Walsh-Hadamard, and Haar transforms [5,6].

Recently, Kronecker and Khatri-Rao (column-wise Kronecker) products have been extensively employed in tensor-based system analysis and modelling, since such products naturally appear in matrix unfoldings of basic tensor decompositions, like the parallel factor (PARAFAC) [7] and Tucker [8] ones, and more generally of constrained PARAFAC models [9]. Reviews of the history and applications of the Kronecker product can be found in [3,5,6,10,11].

In the first part of the paper, we propose a new efficient computational algorithm based on the alternating least-squares (ALS) method for solving the Kronecker product approximation problem, i.e. for determining two matrices **A** and **B** of predetermined sizes,

whose Kronecker product approximates a given matrix **C** in the sense of the minimization of the Frobenius norm $\|\mathbf{C} - \mathbf{A} \otimes \mathbf{B}\|_F$. We also briefly describe four other methods for solving this problem.

During the last decade, tensorial approaches have been widely developed to exploit multiple diversities in wireless communication systems. The principle of diversity techniques is to exploit several copies of the information symbols to be recovered at the receiver. This symbol repetition can result from multipath (due to multiantennas at the transmitter and receiver), repeated transmission of same symbols during several time-slots, and also from specific codings like, for instance, space-time (ST), space-frequency (SF) or space-time-frequency (STF) codings, which induce spatial multiplexing and temporal spreading. Tensor-based multiple-input multiple-output (MIMO) systems allow to improve link reliability as well as to jointly and semi-blindly estimate the channel and the transmitted symbols by means of deterministic receivers operating on data blocks. They have the advantage not to require a priori channel knowledge and long training sequences for estimating the channel. Only very few pilot symbols are needed to eliminate scaling ambiguities inherent to each particular tensor model. Moreover, tensor codings lead to natural tensor formulations of transmitted and received signals, and consequently to tensor system modellings. Tensor-based communication systems can be classified according to:

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- the type of system (code-division multiple access (CDMA), orthogonal frequency division multiplexing (OFDM), CDMA-OFDM);
- the type of coding (ST, STF; matrices/tensors);
- the presence (in [12–16]) or not (in [17–22]) of resource allocation, and their type (matrices/tensors);
- the type of tensor model: PARAFAC [17,18], block PARAFAC [19,20], BTD (block term decomposition) [21], CONFAC (constrained PARAFAC) [12], PARAFAC-Tucker2 (PARATUCK2) [13], PARATUCK-(2,4) [14], generalized PARATUCK [15,16], nested PARAFAC [22].

A brief history of tensor-based systems is now reviewed by beginning with the fundamental work [17], which links direct-sequence CDMA (DS-SS) systems with a PARAFAC model. In [18], a space-time (ST) coding based on a Khatri-Rao (KR) product, denoted KRST, was derived by combining a linear precoding for spatial multiplexing with a linear post-coding for temporal spreading. In [20], the idea of tensor coding was introduced for the first time. A three dimensional tensor allows to combine space-time coding and spatial multiplexing, hence the term space-time multiplexing (STM) coding. The third-order tensor containing the received signals satisfies a block constrained PARAFAC model, with two constraint matrices which depend on the multiplexing parameters.

In [12], a generalized ST spreading scheme was proposed for DS-SS systems, using a precoding tensor which allocates the users' data streams and spreading codes to transmit antennas, by means of three resource allocation matrices. The resulting transmission structure led to a third-order tensor model, called CONFAC, for the received signals. In [13], space-time spreading-multiplexing was proposed by combining a matrix precoding with stream- and antenna-to-slot matrix allocations. The third-order tensor of received signals then satisfies a PARATUCK2 model. In [14], the system of [13] was extended by considering a third-order tensor space-time coding, denoted TST, in order to exploit an extra chip diversity. That leads to a fourth-order PARATUCK-(2,4) model for the received signals tensor.

More recently, tensor approaches have been developed for OFDM, and OFDM-SS systems. In [22], a double Khatri-Rao STF coding, denoted DKRSTF, was proposed for OFDM systems. This coding constitutes an extension of the KRST coding [18], obtained by combining space-frequency pre-coding with time spreading. The received signals form a fourth-order tensor satisfying a nested PARAFAC model. In [15], a spatial coding matrix is combined with two third-order interaction tensors which control a joint time-frequency allocation of data streams and transmit antennas. In [16], the case of OFDM-SS systems is considered. The tensor space-time-frequency (TSTF) coding system was developed with the double objective of increasing the diversity gain by means of a fifth-order coding tensor, which allows to exploit four diversities (space, time, chip, and frequency) at the receiver, and simplifying the resource allocation by using a fourth-order allocation tensor to control the assignment of data streams to transmit antennas in the time-frequency domain. That leads to a generalized PARATUCK model for the fifth-order tensor containing the received signals.

The main contributions of this paper are summarized as follows:

- A new algorithm, called Kronecker-based ALS and denoted KALS, is proposed for solving the Kronecker product approximation problem.
- Eight tensor-based MIMO systems are presented in a unified way, using a generalized PARATUCK model [16].
- A comparative theoretical performance analysis is carried out for the considered tensor-based systems, and the maximal

diversity gain is derived under the assumption of flat or frequency-selective fading channels. The transmission rate and the bandwidth of each system are also given.

- Five new receivers exploiting a Kronecker product of the channel and symbol matrices are derived for jointly estimating these matrices, three being semi-blind and two supervised. A necessary identifiability condition is established for each system.
- Extensive Monte Carlo simulation results are shown to compare the performance of three tensor-based systems, with zero-forcing (ZF) receivers in the case of perfect channel knowledge, on the one hand, and with the KALS receivers for joint semi-blind symbol/channel estimation, on the other hand. Then, the performance of the five proposed Kronecker receivers is compared for the TSTF system.

The rest of the paper is organized as follows. In Section 2, we present the KALS method for solving the Kronecker product approximation problem. Four other algorithms are also described. In Section 3, eight tensor-based systems are presented in a unified way using a generalized PARATUCK tensor model. In Section 4, a comparative theoretical performance analysis is carried out for these systems. Section 5 presents five new Kronecker receivers which use the Kronecker product approximation algorithms introduced in Section 2. Simulation results are shown in Section 6 to illustrate and compare the performance of the STF, TST, and TSTF systems, and also of the five proposed Kronecker receivers for the TSTF system. Finally, Section 7 concludes the paper with some perspectives for future work.

Notations and properties: Scalars, column vectors, matrices, and higher-order tensors are written with lower-case, boldface lower-case, boldface upper-case, and calligraphic letters, i.e. (a , $\mathbf{a}$, $\mathbf{A}$, $\mathcal{A}$), respectively. $\mathbf{A}^T$, $\mathbf{A}^H$, $\mathbf{A}^*$, and $\mathbf{A}^\dagger$ stand for transpose, Hermitian transpose, complex conjugate, and Moore-Penrose pseudo-inverse of $\mathbf{A}$, respectively. $\mathbf{e}_n^{(N)}$ is the n -th canonical basis vector of $\mathbb{R}^N$, $\mathbf{I}_N$ is the identity matrix of order N , $\mathbf{1}_N$ is the $N \times 1$ all-ones column vector, and $\|\cdot\|_F$ is the Frobenius norm. The operator $\text{vec}(\cdot)$ forms a column vector by stacking the columns of its matrix argument, whereas $\text{diag}(\cdot)$ forms a diagonal matrix from its vector argument, and $\text{bdiag}(\mathbf{A}_1, \dots, \mathbf{A}_K)$ forms a block-diagonal matrix with K diagonal blocks. The inverse of the vectorization operator is denoted unvec , so that $\mathbf{x} = \text{vec}(\mathbf{X}) \in \mathbb{C}^J \iff \mathbf{X} = \text{unvec}(\mathbf{x}) \in \mathbb{C}^{I \times J}$.

By convention, the order of dimensions in a product IKJ is linked to the order of variation of the corresponding indices (i, j, k). For instance, given a third-order tensor $\mathcal{X} \in \mathbb{C}^{I \times J \times K}$ with entry $x_{i,j,k}$, its tall mode-1 matrix unfolding $\mathbf{X}_{JK \times I} \in \mathbb{C}^{JK \times I}$ corresponds to a combination of its modes (j, k) such that j varies more slowly than k , implying $x_{i,j,k} = [\mathbf{X}_{JK \times I}]_{(j-1)K+k,i}$.

The Hadamard, Kronecker, and Khatri-Rao products are denoted by $\odot$, $\otimes$, and $\diamond$, respectively. Given $\mathbf{A} \in \mathbb{C}^{I \times J}$, $\mathbf{B} \in \mathbb{C}^{K \times L}$, $\mathbf{C} \in \mathbb{C}^{J \times M}$, $\mathbf{D} \in \mathbb{C}^{L \times N}$, we have

$$(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{A}\mathbf{C}) \otimes (\mathbf{B}\mathbf{D}) \in \mathbb{C}^{IK \times MN}, \quad (1)$$

$$\mathbf{A} \otimes \mathbf{B} = (\mathbf{A}\mathbf{I}_J) \otimes (\mathbf{I}_K \mathbf{B}) = (\mathbf{A} \otimes \mathbf{I}_K)(\mathbf{I}_J \otimes \mathbf{B}), \quad (2)$$

$$\mathbf{A} \otimes \mathbf{B} = (\mathbf{I}_I \mathbf{A}) \otimes (\mathbf{B}\mathbf{I}_L) = (\mathbf{I}_I \otimes \mathbf{B})(\mathbf{A} \otimes \mathbf{I}_L). \quad (3)$$

Given two tensors $\mathcal{A} \in \mathbb{C}^{I_1 \times \dots \times I_N \times J_{N+1} \times \dots \times J_{N+P}}$ and $\mathcal{B} \in \mathbb{C}^{I_1 \times \dots \times I_N \times K_{N+1} \times \dots \times K_{N+Q}}$, of respective orders $N+P$ and $N+Q$, we define the Hadamard product of $\mathcal{A}$ with $\mathcal{B}$, along their common modes $(i_1, \dots, i_N)$, as the tensor $\mathcal{C} = \mathcal{A} \odot_{\{i_1, \dots, i_N\}} \mathcal{B}$ of order $N+P+Q$

whose entries are given by $c_{i_1, \dots, i_N, j_{N+1}, \dots, j_{N+P}, k_{N+1}, \dots, k_{N+Q}} = a_{i_1, \dots, i_N, j_{N+1}, \dots, j_{N+P}} b_{i_1, \dots, i_N, k_{N+1}, \dots, k_{N+Q}}$.

A background with extended bibliography on tensor tools and decompositions, and their applications, is presented in tutorial pa-

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