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$\mathscr{H}_\infty$ state feedback control for MJLS with uncertain probabilities☆

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ABSTRACT

This paper addresses the problem of $\mathscr{H}_\infty$ state feedback control design for discrete-time Markov jump linear systems (MJLS) with uncertain transition probability matrix. The main novelty is that, differently from the existing approaches in the literature, the proposed conditions allow the use of polynomially parameter-dependent Lyapunov matrices to certify the closed-loop stability of the MJLS. Therefore, the method is able to provide $\mathscr{H}_\infty$ controllers in cases where the other techniques fail. The synthesis conditions are given in terms of linear matrix inequality relaxations. Examples illustrate the main advantages of the proposed control design method when compared to other approaches from the literature.

1. Introduction

Various dynamical systems, like economy, manufacturing and aerospace plants, suffer abrupt variations in their structure or parameters for many reasons, including changes in the operating point of a linearized model, component failures, and other similar issues. These plants can be adequately modeled by a class of stochastic hybrid systems named Markov jump linear systems (MJLS). Many theoretical results extending the usual concepts of stability, $\mathscr{H}_2$ and $\mathscr{H}_\infty$ norms have been developed for this class of systems (see Boukas, 2005; Costa, Fragoso, & Marques, 2005 and references therein). An extra motivation for those results comes from networked control systems (NCS) whose packet losses and data delivery features behave similarly to discrete-time MJLS.

In the discrete-time case, each individual operation mode of the MJLS is described by a set of difference equations depending upon a random variable whose evolution is governed by a stochastic process depicted by a Markov chain associated to a transition probability matrix. In practical problems, it can be hard or costly to obtain the exact information about the transition probabilities. For instance, in NCS, random packet dropouts or time delays are difficult to be measured (Li & Shi, 2012). In view of that, the assumption that the probabilities could be affected by uncertain parameters has been incorporated into the models. The pioneer work dealing with uncertain probabilities was probably El Ghaoui and Ait-Rami (1996), where a bounded scalar parameter multiplies the nominal transition probability matrix. Since then, different types of uncertainties affecting the transition probability matrix have been considered, roughly classified in three main groups: partly unknown, when only a few elements are precisely given (Zhang & Boukas, 2009a,b,c; Zhang & Lam, 2010); bounded, when the entries lie within a bounded interval (that is, the available information about the probabilities is inaccurate) (Boukas, 2009; Karan, Shi, & Kaya, 2006); unknown but belonging to a polytope (de Souza, 2006; Oliveira, Vargas, do Val, & Peres, 2009). Concerning the availability of the Markov modes, as pointed out in do Val, Geromel, and Gonçalves (2002), it may be limited by cost or physical accessibility. To overcome this problem, one solution is to design mode-independent or partially mode-dependent controllers or filters (de Souza, Trofino, & Barbosa, 2006; do Val et al., 2002; Liu, Ho, & Sun, 2008).

With respect to the problem of state feedback control design for discrete-time MJLS with uncertain transition probability matrix, a common strategy is to employ parameter-independent Lyapunov matrices, a constant one for each operation mode, to ensure the closed-loop stability of the uncertain MJLS (quadratic stability) (El Ghaoui & Ait-Rami, 1996; Zhang & Boukas, 2009c; Zhang & Lam, 2010). However, a more general class of Lyapunov matrices, depending on the uncertain parameters, can be used to improve the results and to provide a feasible solution when the methods

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based on constant Lyapunov matrices fail, as discussed in de Souza (2006). Although parameter-dependent Lyapunov matrices have been employed to certify closed-loop stability in the problems of $\mathscr{H}_2$ control (Morais, Braga, Oliveira, & Peres, 2013; Oliveira et al., 2009), only parameter-independent Lyapunov matrices have been used for $\mathscr{H}_\infty$ state feedback control (Boukas, 2009; Gonçalves, Fioravanti, & Geromel, 2012; Zhang & Boukas, 2009a). Probably, this is due to the fact that stability and $\mathscr{H}_2$ norm conditions for MJLS have primal and dual versions, allowing a direct linearization through a change of variables, while the bounded real lemma for MJLS has only a primal formulation (Seiler & Sengupta, 2003). This technical communique proposes a new strategy to address the $\mathscr{H}_\infty$ state feedback control problem for discrete-time MJLS. The main novelty of the approach is that polynomially parameter-dependent Lyapunov matrices can be used to assess closed-loop stability with a bound to the $\mathscr{H}_\infty$ norm of the system. Moreover, all types of uncertainties (polytopic, interval and completely unknown entries) can be considered in a systematic way (Morais et al., 2013) by means of the multi-simplex methodology (Oliveira, Bliman, & Peres, 2008). The efficiency and advantages of the proposed approach are illustrated by numerical examples.

2. Problem formulation

Consider the discrete-time MJLS in a fixed complete probability space $(\Omega, \mathscr{F}, \{\mathscr{F}_k\}, \Gamma)$ described as

$$\begin{cases} x(k+1) = A(\theta_k)x(k) + B(\theta_k)u(k) + E(\theta_k)w(k) \\ z(k) = C_z(\theta_k)x(k) + D_z(\theta_k)u(k) + E_z(\theta_k)w(k), \\ w(k) \in \mathscr{L}_2^{n_w}, \quad \mathscr{E}\left(\|x(0)\|^2\right) < \infty, \ k \geq 0 \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^{n_x}$ is the system state, $u(k) \in \mathbb{R}^{n_u}$ is the control input, $w(k) \in \mathbb{R}^{n_w}$ is the noisy input, and $z(k) \in \mathbb{R}^{n_z}$ is the controlled output. The Markov chain $\{\theta_k; k \geq 0\}$ takes values in a finite set $\mathbb{K} \triangleq \{1, \dots, \sigma\}$ with σ modes, such that the jumping process is governed by a transition probability matrix $\Gamma = [p_{ij}]$, with $p_{ij} \geq 0$ and $\sum_{i\in\mathbb{K}} p_{ij} = 1, \ \forall i, j \in \mathbb{K}$, where $p_{ij} = \Pr(\theta_{k+1} = j \mid \theta_k = i), \ \forall k \geq 0$. For ease of notation, whenever possible, θ_k is replaced by $i, \ \forall i \in \mathbb{K}$.

The aim of this paper is to propose a new technique to design an $\mathscr{H}_\infty$ state feedback controller for MJLS with uncertain transition probability matrix and complete, partial or null availability of operation modes[2] such that the resulting closed-loop system is stochastically stable. To this end, it is necessary to present a generalization for the concept of stability applied to MJLS. This definition, named as mean square stability (MSS) (Costa et al., 2005), ensures that $\mathscr{E}[\|x(k)\|] \to 0$ as $k \to \infty$ for any initial condition $x_0 \in \mathbb{R}^{n_x}, \ \theta_0 \in \mathbb{K}$.

The proposed conditions can deal with a transition probability matrix Γ affected by different types of uncertainties. Analogously to Boukas (2009), each probability p_{ij} can lie inside the interval $[\underline{p_{ij}}, \ \overline{p_{ij}}]$ or, as in Zhang and Boukas (2009a,b,c), the elements can be completely unknown, that is, $p_{ij} = ?$. In this case, the minimum and maximum bounds of each unknown element can be inferred, respectively, by $\underline{p_{ij}} = 0$ and the constraint of unit summation of the probabilities in the ith row.

To handle all the different types of uncertainties, this paper employs a systematic procedure[3] performed in two steps. The first step (similarly to Gonçalves et al., 2012 and Morais et al., 2013) models each one of the m uncertain rows of Γ as an uncertain vector belonging to the unit simplex Λ_{N_r}, $r = 1, \dots, m$, given by

$$\Lambda_{N_r} \triangleq \left\{ \zeta \in \mathbb{R}^{N_r} : \sum_{i=1}^{N_r} \zeta_i = 1, \ \zeta_i \geq 0, \ i = 1, \dots, N_r \right\}.$$

For instance, considering

$$\Gamma = \begin{bmatrix} ? & ? \\ [0.3, 0.8] & [0.5, 0.9] \end{bmatrix}, \tag{2}$$

the first row, denoted $p_1(\alpha)$, can be written as a convex combination of the vertices $p_1^{(10)} = [1 \quad 0]$ and $p_1^{(01)} = [0 \quad 1]$, that is, $p_1(\alpha) = \alpha_{11}p_1^{(10)} + \alpha_{12}p_1^{(01)}$, $(\alpha_{11}, \ \alpha_{12}) \in \Lambda_2$. The vertices of the second row $p_2(\alpha)$ are $p_2^{(10)} = [0.3 \quad 0.7]$ and $p_2^{(01)} = [0.5 \quad 0.5]$, such that $p_2(\alpha) = \alpha_{21}p_2^{(10)} + \alpha_{22}p_2^{(01)}$, with $(\alpha_{21}, \ \alpha_{22}) \in \Lambda_2$.

In the second step, following the methodology presented in Morais et al. (2013), the uncertain parameters are combined into one single domain, created by the Cartesian product of m unit simplexes $\Lambda = \Lambda_{N_1} \times \cdots \times \Lambda_{N_m}$, called multi-simplex (Oliveira et al., 2008). The representation of the uncertain probability matrix given by (2), in the multi-simplex domain, yields the following polynomial matrix

$$\begin{aligned}\Gamma(\alpha) &= \alpha_{11}\alpha_{21}\Gamma^{(1010)} + \alpha_{11}\alpha_{22}\Gamma^{(1001)} + \alpha_{12}\alpha_{21}\Gamma^{(0110)} \\ &\quad + \alpha_{12}\alpha_{22}\Gamma^{(0101)}, \qquad \alpha = (\alpha_1, \alpha_2) \in \Lambda,\end{aligned}$$

with matrix coefficients

$$\Gamma^{(1010)} = \begin{bmatrix} 1 & 0 \\ 0.3 & 0.7 \end{bmatrix}, \qquad \Gamma^{(1001)} = \begin{bmatrix} 1 & 0 \\ 0.5 & 0.5 \end{bmatrix},$$

$$\Gamma^{(0110)} = \begin{bmatrix} 0 & 1 \\ 0.3 & 0.7 \end{bmatrix}, \qquad \Gamma^{(0101)} = \begin{bmatrix} 0 & 1 \\ 0.5 & 0.5 \end{bmatrix}.$$

It is important to emphasize that the proposed approach can also be used in the case where the transition probability matrix belongs to a polytopic domain (El Ghaoui & Ait-Rami, 1996; Oliveira et al., 2009).

The $\mathscr{H}_\infty$ norm is defined as an induced energy gain from the input vector $w(k)$ to the to-be-controlled output vector $z(k)$. Generally, this performance index can be computed by a set of coupled algebraic Riccati equations. However, when the MJLS is affected by uncertainties, this strategy can no longer be applied and linear matrix inequality (LMI) conditions become an effective tool to solve this problem (Boukas, 2009; Gonçalves et al., 2012; Zhang & Boukas, 2009a). The next lemma, an extension of Seiler and Sengupta (2003) for the multi-simplex case, presents parameter-dependent LMI conditions associated to an $\mathscr{H}_\infty$ guaranteed cost for an MJLS with uncertain transition probability matrix $\Gamma(\alpha)$.

Lemma 1. *System (1), with B_i and D_{zi} identically null, is MSS and γ is an upper bound to the $\mathscr{H}_\infty$ norm of system (1) if and only if there exist symmetric positive definite parameter-dependent matrices $P_i(\alpha) \in \mathbb{R}^{n_x\times n_x}, \ \forall i \in \mathbb{K}$, such that for each $i \in \mathbb{K}$ and for all $\alpha \in \Lambda$, being $P_{pi}(\alpha) = \sum_{j=1}^{\sigma} p_{ij}(\alpha)P_j(\alpha)$, the parameter-dependent inequalities hold*

$$\begin{bmatrix} A_i & E_i \\ C_{zi} & E_{zi} \end{bmatrix}^T \begin{bmatrix} P_{pi}(\alpha) & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} A_i & E_i \\ C_{zi} & E_{zi} \end{bmatrix} - \begin{bmatrix} P_i(\alpha) & 0 \\ 0 & \gamma^2 I \end{bmatrix} < 0. \tag{3}$$

Lemma 1 presents an infinite dimension problem for the computation of the $\mathscr{H}_\infty$ worst case norm of system (1) since the conditions must be fulfilled for all $\alpha \in \Lambda$. However, as proved in Bliman (2004) and extended in Oliveira et al. (2008) for the multi-simplex domain, such robust (parameter-dependent) LMIs admit a homogeneous polynomial solution of sufficiently large partial degrees, whenever one solution exists.

[2] Note that if a mode-dependent controller is sought, the information about the operation mode must be available in real time.

[3] A Matlab routine is available for download at http://www.dt.fee.unicamp.br/~ricfow/programs/Gamma_Multi_Simplex.m.