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Robust adaptive beamforming with minimum sensitivity to correlated random errors

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ABSTRACT

The standard Capon beamformer is subject to substantial performance degradation in the presence of estimation errors of the signal steering vector and the array covariance matrix. In order to address this problem, robust adaptive beamformers (RABs) have been designed. In this study, we propose a novel RAB from the perspective of the beamformer sensitivity. In particular, we consider the general form of the beamformer sensitivity, implying that the random errors may be not white noise but correlated. Then we suggest to use the inverse of the array sample covariance matrix as the random error covariance. Using this, we propose to compute the Capon beamformer with minimum sensitivity to correlated random errors, considering a Euclidean ball as the uncertainty set for the signal steering vector. Moreover, the Lagrange multiplier methodology can be employed to solve the proposed optimization problem. Numerical results demonstrate the superior performance of the proposed beamformer in the presence of large mismatch relative to other existing approaches such as 'diagonal loading', 'robust Capon' and 'maximally robust Capon' beamformers.

1. Introduction

Capon beamformer is a representative example of adaptive array beamformer which intends to allow the signal of interest (SOI) to pass through without any distortion while the interference signals and noise are suppressed as much as possible, thereby maximizing the output signal-to-interference-plus-noise ratio (SINR). The standard Capon beamformer (SCB) can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}} \mathbf{w} \quad \text{s. t.} \quad \mathbf{w}^H \bar{\mathbf{a}} = 1 \tag{1}$$

with the solution

$$\mathbf{w}_c = \beta_c \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}} \tag{2}$$

where $\bar{\mathbf{a}}$ denotes the nominal SOI steering vector. The immaterial scalar $\beta_c = \frac{1}{\bar{\mathbf{a}}^H \hat{\mathbf{R}}^{-1} \bar{\mathbf{a}}}$ does not affect the array output SINR. The estimated covariance matrix $\hat{\mathbf{R}}$ can be formed by

$$\hat{\mathbf{R}} = \frac{1}{K} \sum_{k=1}^{K} \mathbf{x}(k) \mathbf{x}^H(k) \tag{3}$$

where $\{\mathbf{x}(k)\}_{k=1}^{K}$ denote the array observations or snapshots. Performing eigen-decomposition on $\hat{\mathbf{R}}$ yields

$$\hat{\mathbf{R}} = \hat{\mathbf{U}} \hat{\Gamma} \hat{\mathbf{U}}^H = \sum_{i=1}^{N} \hat{\gamma}_i \hat{\mathbf{u}}_i \hat{\mathbf{u}}_i^H \tag{4}$$

where N is the array sensor number. The matrix $\hat{\mathbf{U}} = [\hat{\mathbf{u}}_1, \ldots, \hat{\mathbf{u}}_N]$ collects all the eigenvectors, and $\hat{\Gamma} = \text{diag}\{\hat{\gamma}_1, \ldots, \hat{\gamma}_N\}$ is a diagonal matrix with the eigenvalues $\hat{\gamma}_1 \geq \cdots \geq \hat{\gamma}_N$ being nonincreasingly ordered. By incorporating knowledge of the white noise variance σ_n^2, we can obtain the maximum likelihood estimation with a noise floor constraint [1]

$$\hat{\mathbf{R}}_{\text{ML}} = \sum_{i=1}^{N} \max\{\hat{\gamma}_i, \sigma_n^2\} \hat{\mathbf{u}}_i \hat{\mathbf{u}}_i^H. \tag{5}$$

It has been pointed out that the SCB may suffer from substantial performance degradations even for small mismatch between the presumed $\bar{\mathbf{a}}$ (or $\hat{\mathbf{R}}$) and its actual value $\mathbf{a}_0$ (or $\mathbf{R}$) [2–5]. This is because in such situation the SOI may be treated as an interference signal and therefore be suppressed erroneously, which is commonly referred to as *signal self-nulling* [10]. In order to address this problem, some robust adaptive beamformers (RABs) have been designed which aim to provide acceptable performance even when the nominal steering vector and covariance matrix depart from their actual values. An excellent review and comparison of the existing robust techniques have been provided in [11,12]; see also the references contained therein.

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The most popular RAB technique is the diagonal loading (DL) method [2] along with its generalized versions [3–5]. The conventional DL beamformer can be formulated as

$$\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}}\mathbf{w} + \xi \mathbf{w}^H\mathbf{w} \quad \text{s. t.} \quad \mathbf{w}^H \bar{\mathbf{a}} = 1. \tag{6}$$

The optimum solution of (6) is given by

$$\hat{\mathbf{w}}_{\mathrm{DL}} = \beta_{\mathrm{DL}}\left(\hat{\mathbf{R}} + \xi\mathbf{I}\right)^{-1}\bar{\mathbf{a}} \tag{7}$$

where $\mathbf{I}$ denotes the identity matrix with appropriate size and the scaling factor $\beta_{\mathrm{DL}} = \frac{1}{\bar{\mathbf{a}}^H\left(\hat{\mathbf{R}}+\xi\mathbf{I}\right)^{-1}\bar{\mathbf{a}}}$ is also immaterial. We can see that the DL method in (6) differs from the SCB of (1) in that an additional term $\xi\mathbf{w}^H\mathbf{w}$ is used, which can be explained by the following fact. When the signal self-nulling occurs, we have $\mathbf{w}^H\mathbf{a}_0 \approx 0$ (where $\mathbf{a}_0$ stands for the true steering vector of the SOI) and simultaneously we also have $\mathbf{w}^H\bar{\mathbf{a}} = 1$ due to the distortionless constraint on the nominal SOI. Consequently, we can obtain the approximation expression $\mathbf{w}^H(\bar{\mathbf{a}} - \mathbf{a}_0) \approx 1$. However, the nominal and actual steering vectors are often close and hence $\| \bar{\mathbf{a}} - \mathbf{a}_0 \|$ is relatively small, where the notation $\|\cdot\|$ represents the Euclidean norm. This implies that the relation $\mathbf{w}^H(\bar{\mathbf{a}} - \mathbf{a}_0) \approx 1$ holds only if $\| \mathbf{w} \|$ is large [13]. Therefore, we use the term $\xi\mathbf{w}^H\mathbf{w}$ in (6) to prevent the norm of the weight vector to become large and in turn avoid signal self-cancellation. In the traditional DL method [2], the loading factor ξ is set in an ad hoc way, typically $\xi = 10\sigma_n^2$ where σ_n denotes the noise power.

The generalized versions of DL [3–5] specifically attempt to establish the relationship between the loading factor and the steering vector uncertainty level. For example, in the RAB presented in [3] it is assumed that the true SOI steering vector belongs to the uncertainty set

$$\mathcal{A}(\varepsilon) \triangleq \{\mathbf{a} | \| \mathbf{a} - \bar{\mathbf{a}} \| \le \varepsilon\} \tag{8}$$

where ε is the pre-selected upper bound on the norm of the steering vector mismatch. Then the RAB in [3] maintains a gain no less than unity within the uncertainty set, while minimizing the output power. That is

$$\begin{aligned} &\min_{\mathbf{w}} \mathbf{w}^H \hat{\mathbf{R}}\mathbf{w} \\ &\text{s. t.} \ |\mathbf{w}^H\mathbf{a}| \ge 1, \quad \forall\, \mathbf{a} \in \mathcal{A}(\varepsilon). \end{aligned} \tag{9}$$

Moreover, it is found that the least gain within the uncertainty set, which corresponds to the worst-case steering vector, has the following form [3]:

$$\min_{\mathbf{a}\in\mathcal{A}(\varepsilon)} |\mathbf{w}^H\mathbf{a}| = |\mathbf{w}^H\bar{\mathbf{a}}| - \varepsilon \| \mathbf{w} \| \ge 1. \tag{10}$$

As a consequence, the problem in (9) can be rewritten in the convex second order cone programming form and be solved efficiently using the interior point method. It is also shown that this RAB technique based on worst-case performance optimization belongs to the diagonal loading approaches. Also, in [4] it is shown that the RABs proposed in [3–5] are equivalent and the essence of them is to replace the nominal SOI steering vector by the vector from the presumed uncertainty set, which results in the maximum output power.

Recently, a novel RAB has been considered in [10] from the perspective of the beamformer sensitivity which is defined as

$$T_{\mathrm{se}} \triangleq \frac{\| \mathbf{w} \|^2}{|\mathbf{w}^H\bar{\mathbf{a}}|^2}. \tag{11}$$

From (10), we observe that the largest deviation of the array gain within the uncertainty set is $\varepsilon \| \mathbf{w} \|$. Therefore, the beamformer sensitivity measures the relative deviation in array response (which is normalized by the uncertainty level ε). Then, the beamforming problem in [10] is formulated as

$$\begin{aligned} &\min_{\mathbf{a},\mathbf{w}} \frac{\| \mathbf{w} \|^2}{|\mathbf{w}^H\bar{\mathbf{a}}|^2} \\ &\text{s. t.} \ \mathbf{w} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a}} \\ &\qquad \mathbf{a} \in \mathcal{A}(\varepsilon). \end{aligned} \tag{12}$$

The basic idea of (12) is to find the vector within the uncertainty set which achieves the minimum beamformer sensitivity. As will be shown subsequently, the assumption of uncorrelated random errors is used implicitly in [10]. In other words, the signals are assumed to be perturbed by the white noise. However, this white noise assumption is not always satisfied [6].

In this paper, we suggest a robust adaptive beamforming which is also based on the beamformer sensitivity. Here we treat a more general case in which the signals are perturbed by correlated random errors. Then we find that it is reasonable to use the inversion of the sample covariance matrix as the random error covariance. We consider a beamformer optimization problem which intends to obtain the minimum beamformer sensitivity to the correlated random errors. Such optimization problem can be solved by the Lagrange multiplier methodology.

2. Proposed robust beamformer

In [2], a general definition of the beamformer sensitivity is given by

$$T_{\mathrm{g-se}} \triangleq \frac{\mathbf{w}^H\mathbf{E}\mathbf{w}}{|\mathbf{w}^H\bar{\mathbf{a}}|^2} \tag{13}$$

where the matrix $\mathbf{E}$ denotes the covariance of the random errors. It is pointed out in [2] that $T_{\mathrm{g-se}}$ is a classic measure of sensitivity to tolerance errors. When the errors are uncorrelated, the covariance becomes the identity matrix and thus the above definition reduces to that in (11). However, this white noise assumption is not always valid. If we take the general form into account, the problem that may arise is the choice of the error covariance matrix. As pointed out in [7,8], a strong priori on the shape of the random errors is seldom the case in practice and the robustness may be endangered if an imprecise covariance matrix is used. In this paper, we suggest to use the inverse of the sample covariance matrix, i.e., $\hat{\mathbf{R}}^{-1}$, to replace the matrix $\mathbf{E}$ in (13), implying that the random errors are assumed to be more related to the subdominant eigenvectors of $\hat{\mathbf{R}}$ than to the dominant eigenvectors. Thus the optimization problem considered in this paper takes the following form:

$$\begin{aligned} &\min_{\mathbf{a},\mathbf{w}} \frac{\mathbf{w}^H\hat{\mathbf{R}}^{-1}\mathbf{w}}{|\mathbf{w}^H\bar{\mathbf{a}}|^2} \\ &\text{s. t.} \ \mathbf{w} = \frac{\hat{\mathbf{R}}^{-1}\mathbf{a}}{\mathbf{a}^H\hat{\mathbf{R}}^{-1}\mathbf{a}} \\ &\qquad \mathbf{a} \in \mathcal{A}(\varepsilon). \end{aligned} \tag{14}$$

The reason of the choice of $\hat{\mathbf{R}}^{-1}$ can also be explained by the following facts.

Let us first rewrite the DL weight vector in (7) as

$$\hat{\mathbf{w}}_{\mathrm{DL}} = \beta_{\mathrm{DL}}\left(\hat{\mathbf{R}} + \xi\mathbf{I}\right)^{-1}\bar{\mathbf{a}} = \beta_{\mathrm{DL}} \sum_{i=1}^{N} \frac{\hat{\mathbf{u}}_i^H\bar{\mathbf{a}}}{\hat{\gamma}_i + \xi}\hat{\mathbf{u}}_i. \tag{15}$$

From (15), we observe that for large eigenvalues the term $\frac{1}{\hat{\gamma}_i+\xi}$ is almost unchanged whether ξ is loaded or not. However, for small eigenvalues the term $\frac{1}{\hat{\gamma}_i+\xi}$ reduces significantly once a positive