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The design of dynamical observers for hybrid systems: Theory and application to an automotive control problem[☆]

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ABSTRACT

A design methodology is presented for dynamical observers of hybrid systems with linear continuous-time dynamics that reconstructs the complete state (discrete location and continuous state) from the knowledge of the inputs and outputs of a hybrid plant. We then present the application of the theory to the problem of on-line identification of the actual engaged gear for a car. The performance of the observer was tested with *experimental data* obtained in a Magneti Marelli Powertrain using an Opel Astra equipped with a Diesel engine.

1. Introduction

Hybrid systems are powerful abstractions for modeling complex systems. Their theoretical properties have been the subject of intense research. In addition, they have been used in a number of applications to provide models better reflecting the nature of control problems such as the ones related to embedded system design where discrete controls are routinely applied to continuous processes. Because of their generality, deriving rigorous controller synthesis procedures is often difficult. In many cases, we must resort either to heuristics or to approximations. Even when the structure of the hybrid problem is such that a controller can be synthesized, strong assumptions have to be used. For example, the use of hybrid formalisms to solve control problems in automotive applications has been proposed and control laws derived (see Balluchi, Di Benedetto, Pinello, & Sangiovanni-Vincentelli, 1999, Balluchi, Di Benedetto, Pinello, Rossi, & Sangiovanni-Vincentelli, 1999, Balluchi et al., 2000, Balluchi, Benvenuti, Di Benedetto, Pinello, & Sangiovanni-Vincentelli, 2000). These hybrid control algorithms are based on full state feedback. However, in most cases, only partial information about the state of the hybrid plant is available. Hence, a method for full state estimation is very important to make hybrid control algorithms really applicable.

A complete theory and a design methodology for observers for mixed logical dynamical systems (MLD) was presented in Bemporad, Ferrari-Trecate, and Morari (2000) (see also Ferrari-Trecate, Mignone, & Morari, 2000). This approach is applicable to any hybrid system that can be approximated by an MLD system and, consequently, it is a general powerful approach. However, even if a hybrid system could be well approximated by an MLD system, when "the observability horizon becomes large, solving the optimization problem can become computationally intractable" (Bemporad et al. (2000)). In Fliess, Join, and Perruquetti (2008), algebraic necessary and sufficient conditions for the distinguishability of a linear switching system are offered. The authors propose a numerically efficient procedure for reconstruction of the state of the system as well as of switching signals and characterize the "bad" inputs to be avoided for which the continuous dynamics are indistinguishable. In Pettersson (2006), switched linear systems without reset and disturbances are considered. The author gives a condition for the existence of an observer based on the existence of a Lyapunov function that is common to the components of the system. In Tanwani, Shim, and Liberzon (2011), observability for the same class of systems is first geometrically characterized. An

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observer is synthesized that generates the state estimate that converges to the actual state under persistent switching.

In this paper we present a design methodology for dynamical observers of hybrid systems with linear continuous-time dynamics that reconstructs the *complete state (discrete location and continuous state) from the knowledge of the hybrid system input and output signals*. The hybrid systems considered here have reset states and disturbances and hence, are more general than the ones addressed in Pettersson (2006); Tanwani et al. (2011). The proposed hybrid observer consists of two parts: a *location observer* that identifies the current location of the hybrid plant, and a *continuous observer* that produces an estimate of the evolution of the continuous state of the hybrid plant. We first introduce the notion of "current-location observable" hybrid systems, for which the sequence of location switching can be identified by the location observer from the observation of the discrete plant output, without the need of any additional information from the evolution of the continuous part of the plant. For this class of hybrid plants, the continuous observer can be designed by exploiting the results on stability of switching systems given in [20] and [13]. We then extend the previous result to the case of not current-location observable hybrid systems by using some additional information obtained from the processing of the continuous plant inputs and outputs for the identification of the sequence of location switchings.

The main contributions of this paper are the following:

- The proposed design methodology allows addressing observer synthesis for hybrid systems with partial discrete information on the current location, spanning the gap between the case of complete knowledge of current location (i.e. the hybrid plant produces as discrete output its current location), treated for instance in Alessandri and Coletta (2001), and the case of absence of any discrete output information (i.e. the hybrid plant produces no discrete output), considered in Bemporad et al. (2000), Ferrari-Trecate et al. (2000). This is an important result for applications where, typically, the discrete actions of the controller are known while no information regarding autonomous switching in the plant is available.
- When the discrete output information is not sufficient to identify the hybrid system location, i.e. the plant is not current-location observable, the processing of the continuous plant input/output signals in the signature generators is independent from the continuous state observation process. This is new with respect to previous approaches (see e.g. Hofbaur & Williams, 2002 and Mosterman & Biswas, 1999) where the estimated values of continuous state of the plant are used to supply the missing information for the identification of the plant location.
- Exponential stabilization of the dynamics of the continuous observation error is obtained by extending the results on stabilization by Morse (1996) and Hespanha and Morse (1999) to the class of switching systems with dwell-time and resets, subject to bounded disturbances.

The proposed hybrid-observer design methodology was applied to an automotive control problem as presented earlier by the authors in Balluchi, Benvenuti, Lemma, Sangiovanni-Vincentelli, and Serra (2005) where the test case was described without the theory that supports the methodology that has been considerably upgraded in this paper. We considered the problem of on-line identification of the actual engaged gear for a car. The relevance of this problem is related to engine control strategies achieving high performance and efficient emissions control which depend critically on the knowledge of the engaged gear. The performance of the observer was tested with experimental data obtained in Magneti Marelli (a tier 1 automotive supplier) using an Opel Astra equipped with a Diesel engine and a SeleSpeed AMT (Automatic Manual Transmission). The signal on actual engaged gear provided by the AMT control unit was used for the validation of the identification algorithm. The specification given by the Magneti Marelli Powertrain Division was to achieve correct identification on a set of maneuvers with a delay of at most 250 msec, using an implementation of the algorithm in discrete-time with a sampling period of 12 msec.

2. Problem formulation

A hybrid system $\mathcal{H}$ is a collection

$$\mathcal{H} = (Q, \Sigma, \Gamma, X, U, W, Y, \mathrm{Init}, f, h, R, \delta, \zeta, \phi)\,, \tag{1}$$

where

- Q, Σ and Γ are the finite sets of discrete state, input and output variables, respectively;
- $X \subseteq \mathbb{R}^n$, $U \subseteq \mathbb{R}^p$, $W \subseteq \mathbb{R}^n$ and $Y \subseteq \mathbb{R}^p$ are the domains of continuous state, input, state disturbance and output variables, respectively;
- $\mathrm{Init} \subseteq Q \times X$ is the set of admissible initial states;
- $f : Q \times X \times U \times W \to TX$ and $h : Q \times X \to Y$ are the vector fields defining the dynamics of the continuous state and output variables and TX is the tangent space to X;
- $R : Q \times Q \times X \to X$ describes the continuous state resets.
- $\delta : Q \times \Sigma \to 2^Q$ and $\zeta : Q \times \Sigma \times Q \to \Gamma \bigcup \{\epsilon\}$ are, respectively, the set-valued functions defining the dynamics of the discrete state and output variables with ϵ being the *null event*;
- $\phi : Q \times X \to 2^{\Sigma \bigcup \{\epsilon\}}$ is the set-valued function specifying the admissible events at each location $q \in Q$, for given values of the continuous state $x(t) \in X$.

The finite set Σ of discrete inputs is composed by both internal events, auto-generated by the hybrid system on the basis of the values of the continuous state $x(t)$, and exogenous input events, whose enabling condition may or may not depend on $x(t)$. The *null event* ϵ is introduced to model different possible transition conditions. For example, if $\phi(q, x) = \{\epsilon\}$, then no input event is enabled for the given value of x while if $\phi(q, x) = \{\sigma, \epsilon\}$, then the input event σ is enabled. Moreover, if $\phi(q, x) = \{\sigma\}$, then the input event σ is forced to occur and this can be used to model internal events forced to occur.

In this paper, we consider *living hybrid systems* defined as those which admit only executions that are non-Zeno and have an infinite number of transitions (see Zhang, Johansson, Lygeros, & Sastry, 2001). For example, viable hybrid systems with 0-lag nonblocking control strategies and initial set Init equal to the viability kernel, are living hybrid systems (see Deshpande & Varaiya, 1994). Moreover, nonblocking hybrid systems with minimum and maximum dwell-time (see De Santis, Di Benedetto, & Pola, 2009) are also living hybrid systems. In the sequel we will consider hybrid systems with no multiple transitions, i.e. hybrid systems for which the times t_k at which discrete transitions take place are such that $t_k < t_{k+1}$.

An execution of a *living hybrid system with no multiple transitions* will involve continuous evolution as well as instantaneous transitions (discrete evolution). In particular:

- $(q(0), x(t_0)) \in \mathrm{Init}$;
- (continuous evolution) for all k, when $t_k \le t < t_{k+1}$,

$$\dot{x}(t) = f(q(k), x(t), u(t), w(t))$$
$$y(t) = h(q(k), x(t))$$
and $\epsilon \in \phi(q(k), x(t))$;