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Short communication

Compressive data gathering with low-rank constraints for Wireless Sensor networks

Jingfei He, Guiling Sun*, Zhouzhou Li, Ying Zhang

College of Electronic Information and Optical Engineering, Nankai University, Tianjin 300071, China

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ABSTRACT

In this paper, a novel compressive data gathering with low-rank constraints is proposed for efficient data gathering and accurate recovery in wireless sensor networks. The proposed method utilizes both the low-rank feature of the data matrix by introducing the historical data and the sparsity feature based on compressive sensing. A reconstruction algorithm based on the alternating direction method of multipliers is described to efficiently solve the resultant optimization problem. Experimental results show the proposed method can significantly improve the recovery accuracy compared with the state-of-the-art methods.

1. Introduction

Wireless sensor networks (WSNs) have been widely used in many applications including military surveillance, environmental monitoring, and healthcare monitoring. Typically, WSNs have one sink and a number of sensor nodes monitoring physical phenomena and communicating among themselves. The sensor nodes sense, process and transmit the data to the sink for processing. This systematic collection of sensed data, termed as data gathering, is one of the key research issues in WSNs.

In order to reduce the number of data packets transmitted in data gathering, a number of methods have been proposed in recent years [1–7]. Most notably, the Compressive Data Gathering (CDG) [2] was proposed to apply Compressed Sensing (CS) [8] to sensor data gathering for large-scale WSNs by exploiting the sparsity feature of the WSNs signal.

Although CDG can collect data and recovery in each sampling time, the recovery accuracy is not satisfying. The method based on CS and incorporating autoregressive (AR) model into the reconstruction (i.e., CDG_AR) [3] was also proposed. And a sequential CS method with sliding window processing (i.e., Seq-Prog-CS) was proposed for reconstruction of spatially and temporally correlated sensor data [9]. Afterwards, another data gathering methods were proposed to exploit the spatiotemporal correlation in the form of low-rankness [4–6]. The method utilizing both the low-rank and temporal sparsity feature was also proposed [7]. These low-rank based methods have been successfully used in practical applications. However, in order to arrange the data into a matrix and utilize the low-rank feature, the sensed data from a number of sampling time slots were needed, which means these low-rank based methods cannot be used in the real-time requirement WSNs.

To improve the recovery accuracy of signal for the real-time requirement WSNs, we proposed a new method utilizing both the low-rank feature of the data matrix by introducing the historical data and the sparsity feature of data from current time slot based on CS.

2. Problem formulation

For simplicity of analysis, consider a WSN with n sensor nodes and one sink. The time is divided into equal-sized time slots. During each time slot, there are n sensor readings generated and defined as an n-dimensional vector $\mathbf{x} \in \mathbb{R}^n$. As the data gathering scheme described in CDG [2], instead of receiving the individual sensor readings, the sink will be sent a few weighted sums of all the readings, which is denoted as $\mathbf{s}$. As a result, the measurement $\mathbf{s} = \mathbf{\Phi}\mathbf{x}$, where $\mathbf{\Phi}$ is an $m \times n$ matrix. The matrix $\mathbf{\Phi}$ maps $\mathbf{x}$ of size n into $\mathbf{s}$ of size m, where m is much smaller than n. Let define $\rho = n/m$ as the compression ratio. Since the sensor readings in the same time slot should be sparse under a certain transform basis $\mathbf{\Psi}$ [2], the problem to recover the original signal $\mathbf{x}$ from $\mathbf{s}$ can be

* Corresponding author.
E-mail address: sungl@nankai.edu.cn (G. Sun).

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expressed as solving an ℓ_0 minimization problem:

$$\min \ \| \boldsymbol{\Psi}\mathbf{x} \|_0 \quad \text{s.t.} \ \boldsymbol{\Phi}\mathbf{x} = \mathbf{s} \tag{1}$$

Unfortunately, the above ℓ_0 minimization problem is NP-hard, and hence solving it requires combinatorial optimization and is impractical. It is equivalent to use minimal ℓ_1 norm representation instead of the ℓ_0 minimization in some sense. Furthermore, by introducing a quadratic penalty term, the optimization problem equation (1) can be converted into a corresponding unconstrained formulation as:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{s} - \boldsymbol{\Phi}\mathbf{x}\|_2^2 + \lambda \|\boldsymbol{\Psi}\mathbf{x}\|_1. \tag{2}$$

3. The proposed method

3.1. Data reconstruction

As described in Section 2, the sensor readings in the current time slot are defined as $\mathbf{x}$. Let define the historical data from last p time slots as $\mathbf{H} \in \mathbb{R}^{n\times p}$, where $\mathbf{H}_{i,j}$ denotes the sensor reading from the ith node at last $(p-j+1)$th time slot. In order to utilize the spatiotemporal correlation often observed in WSNs data, let us combine the historical data and the current data together to obtain the whole data matrix:

$$[\mathbf{H}\ \mathbf{x}] = \begin{bmatrix} \mathbf{H}_{1,1} & \cdots & \mathbf{H}_{1,p} & \mathbf{x}_1 \\ \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_{n,1} & \cdots & \mathbf{H}_{n,p} & \mathbf{x}_n \end{bmatrix} \tag{3}$$

Since the data generated from neighboring sensors in continuous time slots is often redundant and highly correlated [5], the data matrix $[\mathbf{H}\ \mathbf{x}]$ should have low-rank or approximately low-rank property. As a result, the low-rank constraint can be utilized to recover the signal, i.e., minimizing the rank of $[\mathbf{H}\ \mathbf{x}]$. However, it is NP-hard to solve the optimization problem by minimizing the rank of matrix directly [10,11]. Therefore, minimizing the nuclear norm for the matrix is utilized as an alternative [12,13] which can be represented as:

$$\min \ \|[\mathbf{H}\ \mathbf{x}]\|_* \quad \text{s.t.} \ \boldsymbol{\Phi}\mathbf{x} = \mathbf{s}, \tag{4}$$

where the nuclear norm $\|\cdot\|_*$ for the matrix is equal to the sum of the singular values of a matrix. In this paper, we incorporate the sparsity constraint by minimizing the ℓ_1 norm and the low-rank constraint by minimizing the nuclear norm in a single formulation:

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \|\mathbf{s} - \boldsymbol{\Phi}\mathbf{x}\|_2^2 + \lambda \|\boldsymbol{\Psi}\mathbf{x}\|_1 + \mu \|[\mathbf{H}\ \mathbf{x}]\|_*, \tag{5}$$

where λ and μ are the regularization parameters which control the tradeoff between presenting sparsity feature of data from current time slot, achieving low-rank of the whole data matrix, and fitting to the data-fidelity term $\|\mathbf{s} - \boldsymbol{\Phi}\mathbf{x}\|_2^2$.

3.2. Optimization algorithm

Although the constraints imposed by Eq. (5) are appealing from a modeling standpoint, the convex optimization problem with nonsmooth regularization raises issue of computational complexity. The alternating direction method of multipliers (ADMM) [14] can be used due to its suitability for the large-scale and convex optimization problem. Here, we develop an efficient and robust ADMM-based reconstruction algorithm which is simple to implement. The optimization equation in Eq. (5) can be converted into the following equivalent constrained optimization problem through variable splitting,

$$\left\{\hat{\mathbf{x}}, \hat{\mathbf{y}}, \hat{\mathbf{Z}}\right\} = \arg\min_{\mathbf{x},\mathbf{y},\mathbf{Z}} \|\mathbf{s} - \boldsymbol{\Phi}\mathbf{x}\|_2^2 + \lambda \|\mathbf{y}\|_1 + \mu \|\mathbf{Z}\|_*$$
$$\text{s.t.} \ \mathbf{y} = \boldsymbol{\Psi}\mathbf{x}, \quad \mathbf{Z} = [\mathbf{H}\ \mathbf{x}]. \tag{6}$$

Then the augmented Lagrangian function for Eq. (6) can then be written as:

$$\begin{aligned}\mathcal{L}(\mathbf{x}, \mathbf{y}, \mathbf{Z}, \mathbf{a}, \mathbf{B}) &= \|\mathbf{s} - \boldsymbol{\Phi}\mathbf{x}\|_2^2 + \lambda \|\mathbf{y}\|_1 + \langle \mathbf{a}, \mathbf{y} - \boldsymbol{\Psi}\mathbf{x}\rangle + \frac{\alpha}{2} \|\mathbf{y} - \boldsymbol{\Psi}\mathbf{x}\|_2^2 + \mu \|\mathbf{Z}\|_* \\ &\quad + \langle \mathbf{B}, \mathbf{Z} - [\mathbf{H}\ \mathbf{x}]\rangle + \frac{\beta}{2} \|\mathbf{Z} - [\mathbf{H}\ \mathbf{x}]\|_F^2\end{aligned} \tag{7}$$

Here, $\mathbf{a}$ and $\mathbf{B}$ are two Lagrangian multipliers, $\alpha, \beta > 0$ the penalty parameters, $\|\cdot\|_F$ the Frobenius norm. Eq. (7) can be minimized by the following alternating direction method:

$$\mathbf{x}_{k+1} = \arg\min_{\mathbf{x}} \mathcal{L}(\mathbf{x}, \mathbf{y}_k, \mathbf{Z}_k, \mathbf{a}_k, \mathbf{B}_k) \tag{8}$$

$$\mathbf{y}_{k+1} = \arg\min_{\mathbf{y}} \mathcal{L}(\mathbf{x}_{k+1}, \mathbf{y}, \mathbf{a}_k) \tag{9}$$

$$\mathbf{Z}_{k+1} = \arg\min_{\mathbf{Z}} \mathcal{L}(\mathbf{x}_{k+1}, \mathbf{Z}, \mathbf{B}_k) \tag{10}$$

$$\mathbf{a}_{k+1} = \mathbf{a}_k + \alpha(\mathbf{y}_{k+1} - \boldsymbol{\Psi}\mathbf{x}_{k+1}) \tag{11}$$

$$\mathbf{B}_{k+1} = \mathbf{B}_k + \beta(\mathbf{Z}_{k+1} - [\mathbf{H}\ \mathbf{x}_{k+1}]) \tag{12}$$

In practical implementation, $\mathbf{y}_0$ and $\mathbf{a}_0$ were initialized with zeros vectors, and $\mathbf{Z}_0$ and $\mathbf{B}_0$ were all initialized with zeros matrices. The general solutions to the subproblems of Eqs. (8)–(10) are described in Appendix A. The algorithm is terminated when $\|\mathbf{x}_{k+1} - \mathbf{x}_k\|_2 / \|\mathbf{x}_k\|_2$ is smaller than a predefined tolerance parameter, or k exceeds a maximum number of iterations. It is noteworthy that for the convex optimization problem in Eq. (5), the ADMM algorithm is guaranteed to have global convergence from any initializations.

4. Simulation

In order to evaluate the effectiveness of the proposed method for data reconstruction in WSNs, we performed the data collection based on CS and reconstruction experiments with real WSN data.

4.1. Experimental environments

The real WSN data used for experiments was collected from 54 sensors deployed in the Intel Berkeley Research lab [15], where four types of information (e.g., humidity, temperature, light and voltage) were monitored and collected once every 31 s. To implement the proposed method, the compressive data gathering scheme [2] was utilized for the data gathering in WSNs. Specifically, during each time slot, each node calculated the weighted measurement of its reading, and the sink received the sum of weighted measurements. Mathematically, the measurement $\mathbf{s} = \boldsymbol{\Phi}\mathbf{x}$ was generated, where $\boldsymbol{\Phi}$ was a normally distributed random matrix in our simulation.

The size of the measurement $\mathbf{s}$ is $m = \lfloor n/\rho \rfloor$, where ρ is the compression ratio. The CDG [2], CDG_AR [3], Seq-Prog-CS [9], and the proposed method with different p were utilized to reconstruct the signal from the measurement. The orthonormal Fourier matrix was selected as the transform basis $\boldsymbol{\Psi}$ in CDG. CDG_AR, and the proposed method. For Seq-Prog-CS, the discrete Fourier and cosine transform were utilized to form the Kronecker sparsifying basis.