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Unitary transformations for spherical harmonics MUSIC

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ABSTRACT

Spherical arrays have been widely used in direction-of-arrival (DOA) estimation in recent years. In this paper, two unitary transformations for DOA estimation using spherical arrays are developed to transform the complex search vector to a real vector. Furthermore, the covariance matrix is transformed into a real matrix which has real eigenvalues and eigenvectors. Therefore, we can develop a real spherical harmonics multiple signal classification (RSHMUSIC) method to estimate the locations of signals. By exploiting the phase shift characteristic of the real search vector, the computation cost can be further reduced. Compared with the complex spherical harmonics MUSIC, the proposed methods have a lower amount of computations and better performance. Simulation results demonstrate the performance of the real SHMUSIC method.

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1. Introduction

Multiple signal classification (MUSIC) [1] is a popular subspace-based technique for estimating direction-of-arrival (DOA) of incident signals. It decomposes the covariance matrix into signal subspace and noise subspace. Based on the noise subspace and the search vector, the MUSIC spectrum is formed to search the direction space. However, in array signal processing, the search vector is complex and the covariance matrix is a complex Hermitian matrix. Therefore, the eigendecomposition and the calculation of MUSIC search spectrum require complex computations. They need a large amount of computations.

In order to reduce the complexity, a unitary transformation method [2,3] was proposed to transform the complex covariance matrix and the complex search vector into a real symmetric matrix and a real vector, respectively. It saves a significant amount of computations. However, these methods only suit an equally spaced linear array. The unitary transformation method was extended for a uniform rectangular array (URA) [4], two parallel uniform linear arrays (TP-ULAs) [5], and a uniform circular array (UCA) [6], respectively. A spherical array has been widely used in DOA estimation [7–10]. It has a symmetrical structure and can acquire high azimuth and elevation estimation accuracy. Complex-valued MUSIC method has been used to DOA estimation using spherical array [7,8]. It has a large amount of calculations. Kumar et al. proposed the spherical harmonics root-MUSIC [10]. They rewrote the steering vector to illustrate the Vandermonde structure of the array manifold in the spherical harmonics domain. However, this method is only able to get azimuth estimation with the determined elevation. Yan et al. proposed a real-valued MUSIC method [11] for arbitrary linear array geometries because the steering matrix satisfies $\mathbf{A}^{*}(\boldsymbol{\theta}) = \mathbf{A}(-\boldsymbol{\theta})$, where $\boldsymbol{\theta}$ is the signal directions. But the search vector of spherical arrays in the spherical harmonics domain does not have the conjugate property like that of linear arrays, so this method cannot be used for DOA estimation in the spherical harmonics domain. Each element of the real-valued beamspace manifold in [12] contains a sum term in which the degree changes from 0 to the highest order. Therefore, it does not have the independent phase shift property for azimuth or elevation. The computation cost cannot be further reduced. Yan et al. proposed the real-valued time-domain implementation of the broadband modal beamformer in the spherical harmonics domain [13]. The modal transformation is real-valued because the product of the spherical harmonic and its conjugate is real-valued according to the character of spherical harmonics. However, there is no spherical harmonics product in the array signal model (in the following model (6)) for DOA estimation. Therefore, utilizing the property of the steering vector, we want to construct unitary matrices to directly transform the complex steering vector of the model into a real one and then get a real-valued signal model.

In this paper, we develop two unitary transformations for MUSIC using spherical arrays. The array model is firstly constructed in the spherical harmonics domain. The search vector depends on spherical harmonics. Therefore, the conjugate of the element in the search vector has a similar symmetric property as the corresponding spherical harmonic. This can be adopted to find a unitary transformation to make the search vector have a block conjugate symmetric property. Based on this property, the second

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transformation are developed to transform the complex and conjugate symmetric search vector into a real vector. Furthermore, the covariance matrix can be transformed into a real matrix which has real eigenvalues and eigenvectors. Therefore, a real spherical harmonics MUSIC (RSHMUSIC) estimator can be developed to realize the spectral search only using real computations. This decreases the computational cost considerably.

2. Array model in spherical harmonics domain

Let us consider an L-sensor spherical array with the radius R impinged by D narrowband far-field uncorrelated source signals in free field environment. The sensors are omnidirectional and no mutual coupling between them. The elevation and azimuth of the lth sensor are represented by θ_l and ϕ_l, respectively, $\Omega_l = (\theta_l, \phi_l), l = 1, \ldots, L$. The dth source signal $s_d(t)$ with the wavenumber k impinges on the array from the direction $\Phi_d = (\vartheta_d, \varphi_d)$, where ϑ_d and φ_d are the incident elevation and azimuth, respectively, $d = 1, \ldots, D$. The array model can be described as

$$\mathbf{x}(t) = \sum_{d=1}^{D} \mathbf{a}(\Phi_d)s_d(t) + \mathbf{v}(t), \tag{1}$$

where $\mathbf{x}(t) = [x_1(t), \ldots, x_L(t)]^T$ is the array output vector, $\mathbf{v}(t) = [v_1(t), \ldots, v_L(t)]^T$ is the additive white Gaussian noise vector, t is the snapshot index, and $\mathbf{a}(\Phi_d)$ is the dth steering vector which can be represented in spherical harmonics [12,14] as

$$\mathbf{a}(\Phi_d) = \mathbf{Y}(\Omega)\mathbf{B}(k)\mathbf{y}(\Phi_d), \tag{2}$$

where $\mathbf{Y}(\Omega)$ is an $L \times U$ spherical harmonics matrix as follows

$$\mathbf{Y}(\Omega) = \begin{bmatrix} Y_0^0(\Omega_1)Y_1^{-1}(\Omega_1)Y_1^0(\Omega_1)Y_1^1(\Omega_1)\cdots Y_N^N(\Omega_1) \\ Y_0^0(\Omega_2)Y_1^{-1}(\Omega_2)Y_1^0(\Omega_2)Y_1^1(\Omega_2)\cdots Y_N^N(\Omega_2) \\ \vdots\vdots\vdots\vdots\ddots\vdots \\ Y_0^0(\Omega_L)Y_1^{-1}(\Omega_L)Y_1^0(\Omega_L)Y_1^1(\Omega_L)\cdots Y_N^N(\Omega_L) \end{bmatrix}, \tag{3}$$

where N is the highest order of the spherical harmonics, $U = (N+1)^2$, $Y_n^m(\Omega)$ is the spherical harmonic with order n and degree m defined as [15]

$$Y_n^m(\Omega) = Y_n^m(\theta, \phi) = \sqrt{\frac{(2n+1)}{4\pi}\frac{(n-m)!}{(n+m)!}}P_n^m(\cos\theta)e^{im\phi}, \tag{4}$$

where $P_n^m(\cos\theta)$ is the associated Legendre polynomial, $\mathbf{B}(k) = diag\{b_0(kR), b_1(kR), b_1(kR), b_1(kR), \ldots, b_N(kR)\}$, $b_n(kR) = 4\pi i^n j_n(kR)$ for open spherical arrays, $i = \sqrt{-1}$, $j_n(kR)$ is the spherical Bessel function with the order n, and $\mathbf{y}(\Phi_d)$ is a spherical harmonic vector as follows:

$$\mathbf{y}(\Phi_d) = [Y_0^0(\Phi_d), Y_1^{-1}(\Phi_d), Y_1^0(\Phi_d), Y_1^1(\Phi_d), \cdots, Y_N^N(\Phi_d)]^H. \tag{5}$$

Using the N-order inverse spherical Fourier transform (SFT) approximation, the array output vector $\mathbf{x}(t)$ can be denoted as $\mathbf{x}(t) = \mathbf{Y}(\Omega)\mathbf{x}_{nm}(t)$, where $\mathbf{x}_{nm}(t)$ is the array output SFT coefficient vector [15,16]. In the same way, the noise signal can be expressed as $\mathbf{v}(t) = \mathbf{Y}(\Omega)\mathbf{v}_{nm}(t)$, where $\mathbf{v}_{nm}(t)$ is the noise SFT coefficient vector. By combining (1), (2), the inverse SFT representations for $\mathbf{x}(t)$ and $\mathbf{v}(t)$, and using the least squares criterion, we get the model in the spherical harmonics domain as follows

$$\mathbf{x}_{nm}(t) = \sum_{d=1}^{D} \mathbf{a}_{nm}(\Phi_d)s_d(t) + \mathbf{v}_{nm}(t), \tag{6}$$

where $\mathbf{a}_{nm}(\Phi_d) = [a_0^0(\Phi_d), a_1^{-1}(\Phi_d), a_1^0(\Phi_d), a_1^1(\Phi_d), \ldots, a_N^N(\Phi_d)]^T$ is the new steering vector in the spherical harmonics domain with $a_n^m(\Phi_d) = b_n(kR)[Y_n^m(\Phi_d)]^*$.

The spherical harmonics MUSIC (SHMUSIC) spectrum can be expressed as

$$P(\Phi) = \frac{1}{\sum_{u=D+1}^{U}\left|\mathbf{e}_u^H\mathbf{a}_{nm}(\Phi)\right|^2}, \tag{7}$$

where $\mathbf{e}_u$ is the eigenvector corresponding to the uth eigenvalue ($\lambda_1 \geq \cdots \geq \lambda_U$) of the covariance matrix $\mathbf{R} = E[\mathbf{x}_{nm}(t)\mathbf{x}_{nm}^H(t)]$ and $\mathbf{a}_{nm}(\Phi)$ is the search vector [7]. Calculating the MUSIC spectrum requires complex computations because both the eigenvectors and the search vector are complex. In this paper, we develop two unitary transformations to change complex computations into real computations.

3. Unitary transformations for SHMUSIC

3.1. Block centrohermitian matrix

According to [17], a $b \times c$ complex centrohermitian matrix $\mathbf{W}$ satisfies $\mathbf{W} = \mathbf{J}_b\mathbf{W}^*\mathbf{J}_c$, where $\mathbf{J}_b$ represents the $b \times b$exchange matrix with ones on its anti-diagonal positions and zeroes elsewhere.

An $M \times M$ matrix $\mathbf{C}$ can be divided into Γ blocks according to the row index. Each block has p_τ rows, where $\tau = 1, 2, \ldots, \Gamma$ and $\sum_{\tau=1}^{\Gamma} p_\tau = M$. With the same division rule, the matrix can be also divided into Γ blocks according to the column index. Assuming each block of $\mathbf{C}$ is a centrohermitian matrix, a block diagonal exchange matrix can be defined as

$$\bar{\mathbf{J}} = \text{blk}diag(\mathbf{J}_{p_1}, \mathbf{J}_{p_2}, \ldots, \mathbf{J}_{p_\Gamma}). \tag{8}$$

where blkdiag(•) denotes the block diagonal matrix. The block diagonal exchange matrix $\bar{\mathbf{J}}$ satisfies $\bar{\mathbf{J}}\bar{\mathbf{J}} = \mathbf{I}_M$, where $\mathbf{I}_M$ is an $M \times M$ identity matrix.

Definition 1: An $M \times M$ matrix $\mathbf{C}$is called block centrohermitian if

$$\mathbf{C} = \bar{\mathbf{J}}\mathbf{C}^*\bar{\mathbf{J}}. \tag{9}$$

For odd p_τ, define a submatrix as

$$\Xi_{p_\tau} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_{(p_\tau-1)/2} & \mathbf{0}_{(p_\tau-1)/2} & \mathbf{J}_{(p_\tau-1)/2} \\ \mathbf{0}_{(p_\tau-1)/2}^T & \sqrt{2} & \mathbf{0}_{(p_\tau-1)/2}^T \\ -i\mathbf{J}_{(p_\tau-1)/2} & \mathbf{0}_{(p_\tau-1)/2} & i\mathbf{I}_{(p_\tau-1)/2} \end{bmatrix}, \tag{10}$$

where $\mathbf{0}_{(p_\tau-1)/2}$ is a column vector containing $(p_\tau-1)/2$zeros, and for even p_τ, we have

$$\Xi_{p_\tau} = \frac{1}{\sqrt{2}}\begin{bmatrix} \mathbf{I}_{p_\tau/2} & \mathbf{J}_{p_\tau/2} \\ -i\mathbf{J}_{p_\tau/2} & i\mathbf{I}_{p_\tau/2} \end{bmatrix}. \tag{11}$$

Considering all Γ blocks, we can construct a block diagonal matrix as

$$\Xi = \text{blk}diag(\Xi_{p_1}, \Xi_{p_2}, \ldots, \Xi_{p_\Gamma}). \tag{12}$$

It is easily shown that Ξ is unitary, *i.e.*, $\Xi^{-1} = \Xi^H$, and satisfies

$$\Xi^*\bar{\mathbf{J}} = \Xi. \tag{13}$$

Using the unitary matrix Ξ, we have the following theorem.

Table 1
The block division of the estimated covariance matrix according to the row index.

Order n	0	1	2	3	4
Block index	1	2	3	4	5
Row index	1	2–4	5–9	10–16	17–25

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