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Phase correction autocorrelation-based frequency estimation method for sinusoidal signal

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ABSTRACT

To improve the precision of frequency estimation, a phase correction autocorrelation-based frequency estimation method for sinusoidal signal is proposed. Firstly, a phase correction autocorrelation is developed to reduce the effect of non-half period sampling signal on autocorrelation. Secondly, reference signal is generated according to phase correction autocorrelation signal. Finally, an error function between phase correction autocorrelation signal and reference signal is constructed and frequency estimation is obtained by calculating the minimum of error function. To demonstrate the superiority of the proposed method, computational complexity is analyzed, simulations and experiments are performed. Theoretical analysis and simulations demonstrate that the proposed method reduces the influence of non- half period sampling signal and has better frequency estimation performance than the interpolated DFT method, the modified covariance method for correlation, the two-stage autocorrelation method and the expanded autocorrelation method. The measurement experiments of LFMCW radars validate the effectiveness and superiority of the proposed method in practice.

1. Introduction

Frequency estimation of sinusoidal signal has received much attention in the literature because of its wide application in numerous engineering applications, such as radar, sonar, communication, power systems, measurement and instrumentation [1–5]. For example, the measurement accuracy of linear frequency modulation continuous wave (LFMCW) radars depends on frequency estimation of LFMCW radar signal directly. Therefore, accurate frequency estimation of real sinusoid is of great significance for practical engineering applications.

To obtain frequency estimation of sinusoidal signal, many digital methods are presented during the last decades. The existing frequency estimation methods are mainly divided into two categories: the frequency-domain methods and the time-domain methods. The frequency-domain methods [6,7] transform sampled signal from time domain to frequency domain by discrete Fourier transform(DFT) and obtain frequency estimation through discrete spectrum correcting of DFT coefficients. The methods are computationally simple and have good anti-interference performance. However, the frequency-domain methods suffering from spectral leakage have difficulty in obtaining unbiased frequency estimation when dealing with real sinusoid with a finite signal length [8]. The time-domain methods obtain estimated frequency by means of autocorrelation, linear prediction and so on. As one of the time-domain methods, the Pisarenko harmonic decomposition (PHD) method [9,10] exploits the eigenstructure of sampled signal's covariance matrix to gain its estimated frequency. Despite it is easily implemented, the PHD method is sensitive to the noise [11], which limits its application in practice. To improve the estimation performance of the PHD method, the modified Pisarenko harmonic decomposition (MPHD) method [11] and the reformulation of Pisarenko harmonic decomposition (RPHD) method [12] are developed. Compared with the PHD method, the RPHD and MPHD methods have better estimation performance. To improve the anti-interference performance of the PHD methods, the modified covariance method for correlation [13,14] (MCC) and the expanded autocorrelation(EA) method [15] make full use of multiple autocorrelation signals to calculate frequency estimation. The MCC and EA methods improve the frequency estimation precision at low SNR effectively. But their performance shows no improvement at median or high SNR corresponding to the increasing SNR, which results from non-half period sampling of sinusoidal signal. The two-stage autocorrelation (TSA) method [16] gains frequency estimation by means of two-stage autocorrelation of sampled signal. The TSA method achieves asymptotically unbiased frequency estimation and improves its estimation performance, but it involves more extensive computational complexity and has relatively poor

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performance at low SNR, which is attributed to two-stage autocorrelation of sampled signal [15].

To improve the frequency estimation performance of real sinusoid, a phase correction autocorrelation-based frequency estimation method for sinusoidal signal is developed. The rest of the paper is organized as follows. In Section 2, the key idea of the proposed method is introduced. The phase correction autocorrelation is devised to reduce the influence of non-half period sampling of sinusoidal signal and the error function reflecting the initial phase and frequency matching degree between autocorrelation signal and reference signal is constructed. Besides, the basic step of this method is given and computer simulations are implemented to demonstrate the superiority of phase correction autocorrelation. Computational complexity is analyzed in Section 3. To assess the frequency estimation performance of the proposed method, simulations and field experiments are conducted in Section 4 by comparing with the interpolated DFT (IDFT), MCC, TSA and EA methods and the Cramér-Rao lower bound (CRLB). Finally, conclusions are outlined in Section 5.

2. Method development

The discrete-time signal model for real single-tone sinusoidal signal with Nsamples can described as

$$x(n) = s(n) + z(n),\ n = 1, 2\cdots N \tag{1}$$

where $s(n) = a\cos(n\omega + \theta)$ is noise-free sinusoidal signal, the $a > 0$, $\omega \in (0, \pi)$ and $\theta \in \left[0, 2\pi\right)$ are unknown but deterministic constants which represent the sinusoidal magnitude, frequency, and initial phase, respectively. $z(n)$ is zero-mean, white Gaussian noise with variance σ^2.

To obtain frequency estimation of sampled signal, the kth autocorrelation signal of sampled signal is calculated as follows:

$$R_o(k) = \sum_{n=1}^{N-k} x(n)x(n+k) = \frac{a^2}{2}(N-k)\cos(k\omega) + \varepsilon_o(k) + v_o(k) \tag{2}$$

where $k = 1, 2\cdots N-1$, $v_o(k) = \sum_{n=1}^{N-k}\left[s(n)z(n+k) + z(n)s(n+k) + z(n)z(n+k)\right]$ represents cross-correlation function value of the noise at different times, the noise and noise-free signal. The error term, $\varepsilon_o(k)$ is

$$\varepsilon_o(k) = \sum_{n=1}^{N-k} \frac{a^2}{2}\cos\left((2n+k)\omega + 2\theta\right) \tag{3}$$

Due to the fact that the noise at different times is uncorrelated and the noise and noise-free signal are uncorrelated, the expectation of $R_o(k)$ is

$$E\left[R_o(k)\right] = \frac{a^2}{2}(N-k)\cos(k\omega) + \varepsilon_o(k) \tag{4}$$

Therefore, the noise is neglected when N is sufficiently large.

It is evident in Eq. (4) that when $N-k$, the length of two signals $x(n)$ and $x(n+k)(n = 1, 2\cdots N-k)$ used to calculate $R_o(k)$ meets the requirement of $(N-k)\omega = q_1\pi$ (q_1 is an integer), $\varepsilon_o(k)$ equals zero. Therefore, when signal length of $x(n)$ $(n = 1, 2\cdots N-k)$, $N-k$ satisfies the requirement of $(N-k)\omega = q_1\pi$ (q_1 is an integer), the signal $x(n)$ is called as half period sampling signal, that is, $x(n)$ is of half period sampling. On the contrary, $x(n)$ is called as non-half period sampling signal, that is, $x(n)$ is of non-half period sampling.

When $\varepsilon_o(k)$ equals zero, $R_o(k)$ can be simplified as

$$R_o(k) = \frac{a^2}{2}(N-k)\cos(k\omega) \quad k = 1, 2\cdots K,\ K = N-1 \tag{5}$$

At this time, the initial phase of $R_o(k)$ is zero and the phase of $R_o(k)$ is a linear function of unknown frequency ω. When $\varepsilon_o(k)$ does not equal zero, there is a deviation between $R_o(k)$ and $\frac{a^2}{2}(N-k)\cos(k\omega)$. At the time, the initial phase of $R_o(k)$ is not zero and the phase of $R_o(k)$ is a combination of unknown frequency ω and initial phase θ, which results in the difficulty of obtaining accurate frequency estimation based on autocorrelation signal $R_o(k)$.

To suppress the effect of non-half period sampling on autocorrelation, $R(k)$ is calculated as follows:

$$R(k) = R_o(k) - x(1)x(1+k) - x(N-k)x(N) \tag{6}$$

That is

$$R(k) = \sum_{n=2}^{N-k-1} x(n)x(n+k) = \frac{a^2}{2}(N-k-2)\cos(k\omega) + \varepsilon(k) + v(k) \tag{7}$$

where $k = 1, 2\cdots K$, $K = N-3$, $v(k) = \sum_{n=2}^{N-k-1}\left[s(n)z(n+k) + z(n)s(n+k) + z(n)z(n+k)\right]$ represents cross-correlation function value of the noise at different times, the noise and noise-free signal, $\varepsilon(k)$ is

$$\varepsilon(k) = \sum_{n=2}^{N-k-1} \frac{a^2}{2}\cos\left((2n+k)\omega + 2\theta\right) \tag{8}$$

The expectation of $R(k)$ is

$$E\left[R(k)\right] = \frac{a^2}{2}(N-k-2)\cos(k\omega) + \varepsilon(k) \tag{9}$$

To reduce the effect of $\varepsilon(k)$ on autocorrelation and frequency estimation, a phase correction factor $\varepsilon'(k)$ is designed according to (Eqs. (7) and 8).

$$\varepsilon'(k) = \sum_{n=2}^{N-k-1} x(n-1)x(n+k+1) - \frac{a^2}{2}\left(N-k-2\right)\cos\left((k+2)\omega\right) - v'(k) \tag{10}$$

where $v'(k) = \sum_{n=2}^{N-k-1}\left\{s(n-1)z(n+k+1) + z(n-1)\left[s(n+k+1) + z(n+k+1)\right]\right\}$, represents cross-correlation function value of the noise at different times, the noise and noise-free signal.

Due to the fact that

$$\sum_{n=2}^{N-k-1} x(n-1)x(n+k+1) = \sum_{n=1}^{N-k-2} x(n)x(n+k+2) = R_o(k+2) \tag{11}$$

There is

$$\varepsilon'(k) = R_o(k+2) - \frac{a^2}{2}\left(N-k-2\right)\cos\left((k+2)\omega\right) - v'(k) \tag{12}$$

Similar to $v(k)$, $v'(k)$ approximately equals zero when N is sufficiently large. Thus, $v'(k)$ is neglected on the condition.

Due to $E\left[\varepsilon'(k)\right] = \varepsilon(k)$, substituting $\varepsilon(k)$ with $\varepsilon'(k)$ in Eq. (7) and neglecting the influence of $v(k)$ and $v'(k)$ yields

$$R(k) = \frac{a^2}{2}(N-k-2)\cos(k\omega) + R_o(k+2) - \frac{a^2}{2}\left(N-k-2\right)\cos\left((k+2)\omega\right) \tag{13}$$

According to Eq. (13), Eq. (14) is obtained.

$$R(k) - R_o(k+2) = \frac{a^2}{2}(N-k-2)\cos(k\omega) - \frac{a^2}{2}\left(N-k-2\right)\cos\left((k+2)\omega\right) \tag{14}$$