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Short communication

A shrinkage variable step size for normalized subband adaptive filters

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ABSTRACT

The conventional normalized subband adaptive filter (NSAF) using a constant step-size generally faces an inherent trade-off between the steady-state misalignment and the convergence rate. We propose herein a variable step-size NSAF algorithm by minimizing the mean-square deviation (MSD) between the optimal weight vector and the weight vector estimate with the utilization of the shrinkage denoising technique. With the estimation error involved in the step-size adaptation for each subband individually, the proposed algorithm is capable of tracking non-stationary environments. Without the explicit whitening assumption of the input signal in each subband, the proposed algorithm exhibits low steady-state MSD even when the input signal of each subband is colored. Simulation results validate the low misalignment and good tracking ability of the proposed algorithm in system identification application.

1. Introduction

The celebrated least-mean-square (LMS) and the normalized least-mean-square (NLMS) algorithms are widely used in many applications such as channel estimation, echo cancellation and system identification due to their simplicity and robustness. However, the LMS-type algorithms generally suffer from slow convergence rate when the input signal samples are highly correlated. One of the solutions to this problem is the normalized subband adaptive filter (NSAF) recently originated in [1]. The NSAF algorithm uses analysis filters to partition the input signal into multiple subbands, so that the input signal in each subband is whitened. However, since the NSAF uses a constant step-size, there is an inherent trade-off between steady-state misalignment and convergence rate, namely, small step-sizes improve steady-state performance at the cost of convergence slow-down.

To tackle this dilemma, several variable step-size algorithms have been proposed recently to enhance the performance of the NSAF. In [2], a variable step-size matrix NSAF (VSSM-NSAF) has been proposed and is capable of tracking non-stationary systems. This algorithm, however, exhibits a large steady-state misalignment. Based on the minimization of the mean-square deviation (MSD), two recently proposed variable step-size NSAF algorithms, the NSAF-VSS [3] and the V-NSAF [4], exhibit good performance of misalignment and convergence rate. However, since the NSAF-VSS [3] is designed for stationary systems, its tracking ability remarkably degrades for varying environment, while the V-NSAF [4] could intrinsically track the environment changes, due to the fact that its step-size update formula is a function of the estimation error. More recently, the NSAF-VSS [3] has been generalized by adapting the step-size for each individual subband (VISS-NSAF [5]). The VISS-NSAF could achieve faster convergence rate and smaller steady-state estimation error than the NSAF-VSS algorithm, but it still cannot track the variation of the environment well.

Furthermore, it should be noted that both the aforementioned V-NSAF and NSAF-VSS algorithms assume that the input signal in each subband is close to be white. As a consequence, when the input signal samples are highly correlated, increased subbands are in demand to ensure the rationality of whitening assumption in the NSAF structure, which, in turn, results in increased cost of analysis/synthesis filters.

We propose herein an optimal step-size scheme for the NSAF based on minimizing the MSD at each iteration without the assumption of whiteness of the subband input signal. We further develop a sub-optimal variable step-size algorithm with the utilization of the shrinkage denoising technique[1]. As validated by illustrative simulation results, when the input signal is highly correlated, the proposed algorithm exhibits improved performance, compared with the aforementioned VSSM-NSAF [2], NSAF-VSS [3], V-NSAF [4] and VISS-NSAF [5] algorithms as well as the original

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[1] We were informed that the proposed methodology herein was independently discovered by Yu, Zhao and Chen [6].

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NSAF algorithm [1] in terms of both steady-state misalignment and tracking ability. The proposed step-size scheme may be further utilized to enhance the performance of the sign subband adaptive filters (SSAFs) [7–9], which are demonstrated to be robust for impulsive interference.

The rest of this work is organized as follows. In Section 2, the conventional NSAF algorithm is briefly reviewed. In Section 3, the optimal step-size algorithm is derived, and the shrinkage-based variable step-size method is developed in Section 4. Then, the simulation validation of the proposed algorithm is given in Section 5, and the conclusions are given in Section 6.

2. The NSAF algorithm

The structure of the NSAF is illustrated in Fig. 1, where N is the number of subbands. The desired signal $d(n)$ originates from an unknown system

$$d(n) = \mathbf{w}_{opt}^H \mathbf{u}(n) + \eta(n) \tag{1}$$

where $\mathbf{w}_{opt} = [w_0, w_1, w_2, \ldots, w_{M-1}]^T$ denotes the M-dimensional unknown complex-valued coefficient vector to be estimated, $\mathbf{u}(n) = [u(n), u(n-1), \ldots, u(n-M+1)]^T$ is the complex-valued input vector, $\eta(n)$ is the zero-mean Gaussian measurement noise, and the superscripts T and H denote vector/matrix transposition and complex conjugate transposition, respectively.

The signals $d(n)$, $\mathbf{u}(n)$, $\eta(n)$ are partitioned into $d_i(n)$, $\mathbf{u}_i(n)$, $\eta_i(n)$ through the analysis filters $H_i(z)$, $i = 0, 1, \ldots, N-1$. Note that we use n and k to index the original signal sequences and the decimated sequences, respectively. Thus, the ith subband output signal shown in Fig. 1 is defined as $y_{i,D}(k) \triangleq \hat{\mathbf{w}}^H(k)\mathbf{u}_i(k)$, where $\mathbf{u}_i(k) = [u_i(kN), u_i(kN-1), \ldots, u_i(kN-M+1)]^T$ is the ith subband input signal. Furthermore, $d_{i,D}(k)$ and $y_{i,D}(k)$ are generated by decimating $d_i(n)$ and $y_i(n)$, respectively.

The update equation for the conventional NSAF [1] is

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \mu \sum_{i=0}^{N-1} \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}^*(k) \tag{2}$$

where μ is the step size, $\|\cdot\|$ and the superscript $*$ denote the $\mathcal{L}_2$ norm and complex conjugation, respectively. Note that $\hat{\mathbf{w}}(k)$ is the fullband adaptive weight vector. The estimation error signal $e_{i,D}(k)$ of the ith subband is given by

$$e_{i,D}(k) = d_{i,D}(k) - \hat{\mathbf{w}}^H(k)\mathbf{u}_i(k), \quad i = 0, 1, \ldots, N-1 \tag{3}$$

where $d_{i,D}(k) = \mathbf{w}_{opt}^H(k)\mathbf{u}_i(k) + \eta_{i,D}(k)$, and $\eta_{i,D}(k)$ is the ith subband's noise.

3. The optimum step-size for the nsaf

To develop the variable step-size algorithm, we now replace the constant step-size μ in (2) by variable step-sizes $\mu_i(k)$, $i \in 0, 1, \ldots, N-1$ for each subband as below

$$\hat{\mathbf{w}}(k+1) = \hat{\mathbf{w}}(k) + \sum_{i=0}^{N-1} \mu_i(k) \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}^*(k). \tag{4}$$

To deduce the optimal step-size, we require minimizing the MSD, $\psi(k) \triangleq E\{\|\boldsymbol{\varepsilon}(k)\|^2\}$, where the weight error vector $\boldsymbol{\varepsilon}(k)$ is defined as $\boldsymbol{\varepsilon}(k) \triangleq \mathbf{w}_{opt} - \hat{\mathbf{w}}(k)$, and $E\{\cdot\}$ denotes the mathematical expectation operator. With (4) subtracting from $\mathbf{w}_{opt}$ and some direct mathematical operations, we have

$$\boldsymbol{\varepsilon}(k+1) = \boldsymbol{\varepsilon}(k) - \sum_{i=0}^{N-1} \mu_i(k) \frac{\mathbf{u}_i(k)}{\|\mathbf{u}_i(k)\|^2} e_{i,D}^*(k). \tag{5}$$

Note that the derivations of both the NSAF-VSS [3] and V-NSAF algorithms [4] are based on the essential assumption that each subband input is close to be white, i.e., $\mathbf{u}_i(k)\mathbf{u}_i^H(k) \approx \sigma_{u_i}^2(k)\mathbf{I}_M$ and $\mathbf{u}_i^H(k)\mathbf{u}_i(k) \approx M\sigma_{u_i}^2(k)$, $i = 0, 1, \ldots, N-1$, where $\sigma_{u_i}^2(k)$ is the variance of the ith subband input signal, and $\mathbf{I}_M$ denotes the identity matrix of M dimension. Practically, however, it could be difficult or prohibitively expensive to partition strongly correlated signals into whitened subband signals using analysis filters. Thus the performance of the algorithms would likely deteriorate in these scenarios.

We herein propose a variable step-size algorithm based on the

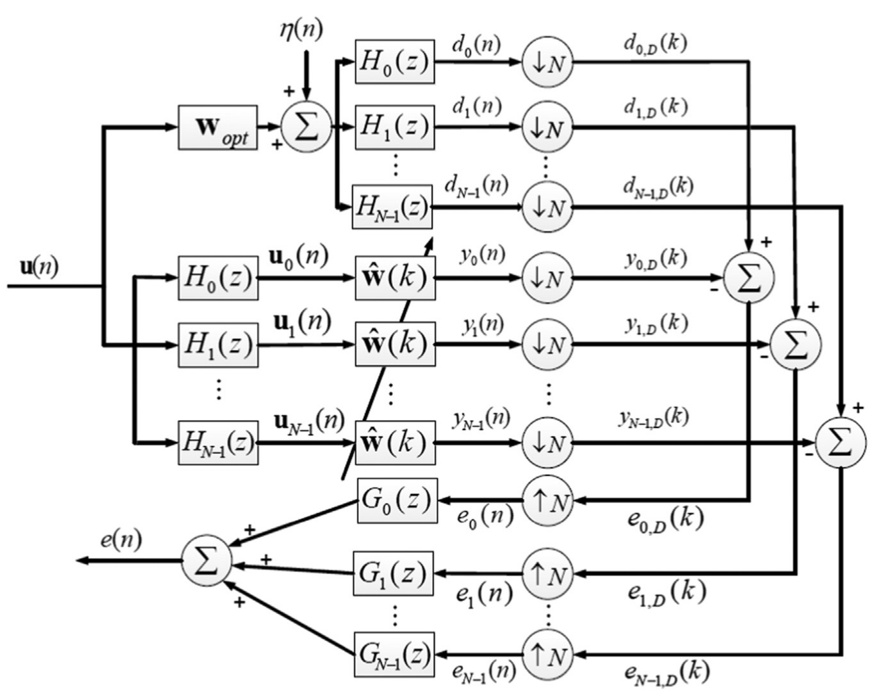

Fig. 1. Structure for the conventional normalized subband adaptive filter.