Automatica 50 (2014) 1632–1640

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

Realization and identification of autonomous linear periodically time-varying systems☆

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ARTICLE INFO

Article history:
Received 5 February 2013
Received in revised form
4 March 2014
Accepted 10 March 2014
Available online 18 April 2014

Keywords:
Linear periodically time-varying systems
Lifting
Realization
Kung's algorithm
Hankel low-rank approximation
Maximum likelihood estimation

ABSTRACT

The subsampling of a linear periodically time-varying system results in a collection of linear time-invariant systems with common poles. This key fact, known as "lifting", is used in a two-step realization method. The first step is the realization of the time-invariant dynamics (the lifted system). Computationally, this step is a rank-revealing factorization of a block-Hankel matrix. The second step derives a state space representation of the periodic time-varying system. It is shown that no extra computations are required in the second step. The computational complexity of the overall method is therefore equal to the complexity for the realization of the lifted system. A modification of the realization method is proposed, which makes the complexity independent of the parameter variation period. Replacing the rank-revealing factorization in the realization algorithm by structured low-rank approximation yields a maximum likelihood identification method. Existing methods for structured low-rank approximation are used to identify efficiently a linear periodically time-varying system. These methods can deal with missing data.

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1. Introduction

1.1. Overview of the literature

Periodically time-varying systems, i.e., systems with periodic coefficients, appear in many applications and have been studied from both theoretical and practical perspectives. The source of the time-variation can be rotating parts in mechanical systems Bittanti and Colaneri (2008); hearth beat and/or breathing in biomedical applications Ionescu, Kosinski, and De Keyser (2010); Sanchez et al. (2013); and seasonality in econometrics (Ghysels, 1996; Osborn, 2001). Linear periodically time-varying systems also appear when a nonlinear system is linearized about a periodic trajectory Sracic and Allen (2011).

In this paper, we restrict our attention to the subclass of discrete-time autonomous linear periodically time-varying systems. A specific application of autonomous linear periodically time-varying system identification in mechanical engineering is vibration analysis, also known as operational modal analysis; see, e.g., Allen and Ginsberg (2006) and Allen, Sracic, Chauhan, and Hansen (2011). The problems considered in the paper are exact (Section 2, Problem 1) and approximate (Section 5, Problem 2) identifications. The exact identification of an autonomous linear periodically time-varying system is equivalent to realization of an input–output linear periodically time-varying system from impulse response measurement. The approximate identification problem yields a maximum-likelihood estimator in the output error model.

Input–output identification methods for linear periodically time-varying systems are proposed in Hench (1995); Liu (1997); Mehr and Chen (2002); Verhaegen and Yu (1995); Xu, Shi, and You (2012) and Yin and Mehr (2010). Less attention is devoted to the autonomous identification problem. A method for exact identification, based on polynomial algebra, is proposed in Kuijper (1999) and a frequency domain method for output-only identification is developed in Allen (2009) and Allen and Ginsberg (2006). Both the method of Kuijper (1999); Kuijper and Willems (1997) and the method of Allen (2009) are based on a lifting approach, i.e., the time-varying system is represented equivalently as a multivariable time-invariant system. The number of outputs p' of the lifted system is equal to the number of outputs p of the original periodic system times the number of samples P in a period of the parameter variation.

☆ The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Alessandro Chiuso under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2014.04.003
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$$\begin{array}{c} \text{data} \\ y \end{array} \xrightarrow{\text{lifting}} \begin{array}{c} \text{lifted} \\ \text{data } y' \end{array} \xrightarrow[\text{realization/identification}]{\text{LTI system}} \begin{array}{c} \text{LTI model} \\ \text{for } y' \end{array} \xrightarrow[\text{computation}]{\text{LPTV parameter}} \begin{array}{c} \text{LPTV model} \\ \text{for } y \end{array}$$

Fig. 1. Main steps of the linear periodic time-varying system realization/identification methods. (LTI—linear time-invariant, LPTV—linear periodically time-varying).

1.2. Aim and contribution of the paper

Most methods proposed in the literature consist of the following main steps (see also Fig. 1):

(1) *preprocessing*—lifting of the data,
(2) *main computation*—derivation of a linear time-invariant model for the lifted data,
(3) *postprocessing*—derivation of an equivalent linear periodically time-varying model.

The key in solving the linear periodic time-varying realization and identification problem is the lifting operation, which converts the time-varying dynamics into time-invariant dynamics of a system with $\mathtt{p}' = \mathtt{p}P$ outputs. From a computational point of view, the realization of the lifted dynamics is a rank-revealing factorization of a block-Hankel matrix. A numerically stable way of doing this operation is the singular value decomposition of a $\mathtt{p}'L \times (T - L)$ matrix, where L is an upper bound on the order, $\mathtt{p}$ is the number of outputs, and T is the number of time samples. Its computational complexity is $O(L^2\mathtt{p}^2PT)$ operations.

Once the linear time-invariant dynamics of the lifted model is obtained, it is transformed back to a linear periodically time-varying model in a postprocessing step. In the subspace identification literature, see, e.g., Hench (1995), this operation is done indirectly by computing shifted versions of the state sequence of the model and solving linear systems of equations for the model parameters. This method, referred to as the "indirect method" is Algorithm 1 in the paper, has computational complexity $O(L^2\mathtt{p}^2P^2T)$.

The main shortcoming of the indirect method is that it requires extra computations for the derivation of the shifted state sequences and the solution of the systems of equations for the model parameters. This increases the computational complexity by a factor of P compared with the complexity of the realization of the lifted system. We show in Section 4 that the linear periodically time-varying model's parameters can be obtained directly from the lifted model's parameters without extra computations. The resulting method, referred to as the "direct method," is Algorithm 2 in the paper. Its computational complexity is $O(L^2\mathtt{p}^2PT)$. A further improvement of the indirect method (Algorithm 3) operates on a $L \times \mathtt{p}(T - L)$ Hankel matrix and requires $O(L^2\mathtt{p}T)$ operations.

The maximum-likelihood estimation problem is considered in Section 5. Using the results relating the realization problem to rank revealing factorization of a Hankel matrix constructed from the data, we show that the maximum-likelihood identification problem is equivalent to Hankel structured low-rank approximation. Subsequently, we use existing efficient local optimization algorithms Usevich and Markovsky (2013) for solving the problem.

The motivation for reformulating the maximum likelihood identification problem as structured low-rank approximation is the possibility to use readily available solution methods. Structured-low-rank approximation is an active area of research that offers a variety of solution methods, e.g., convex relaxation methods, based on the nuclear norm heuristic. There are also methods for solving problems with missing data Markovsky and Usevich (2013). Identification with missing data is a challenging problem; however, using the link between system identification and low-rank approximation, the identification of autonomous periodically time-varying systems with missing data becomes merely an application of existing methods.

The main contributions of the paper are summarized next.

(1) Reduction of the computational cost of linear periodically time-varying system realization from $O(L^2\mathtt{p}^2P^2T)$ to $O(L^2\mathtt{p}T)$.
(2) Maximum-likelihood method for linear periodically time-varying system identification with computational complexity per iteration that is linear in the number of data points. In addition, the maximum-likelihood method can deal with missing data.

2. Preliminaries, problem formulation, and notation

An autonomous discrete-time linear time-varying system $\mathscr{B}$ can be represented by a state space model

$$\mathscr{B} = \mathscr{B}(A, C) := \{y \mid x(t+1) = A(t)x(t),\, y(t) = C(t)x(t), \text{ for all } t, \text{ with } x(1) = x_{\text{ini}} \in \mathbb{R}^{\mathtt{n}}\}, \tag{1}$$

where $A(t) \in \mathbb{R}^{\mathtt{n}\times\mathtt{n}}$ and $C(t) \in \mathbb{R}^{\mathtt{p}\times\mathtt{n}}$ are the model coefficient functions—A is the state transition matrix and C is the output matrix. A state space representation $\mathscr{B}(A, C)$ of the model $\mathscr{B}$ is not unique due to a change of basis, i.e.,

$$\mathscr{B} = \mathscr{B}(A, C) = \mathscr{B}(\widehat{A}, \widehat{C}),$$

where, for all t

$$\widehat{A}(t) = V(t+1)A(t)V^{-1}(t) \quad \text{and} \quad \widehat{C}(t) = C(t)V^{-1}(t), \tag{2}$$

with a nonsingular matrix $V(t) \in \mathbb{R}^{\mathtt{n}\times\mathtt{n}}$.

In this paper, we consider the subclass of autonomous linear time-varying systems, for which the coefficient functions A and C are periodic with period P

$$A(t) = A(t+kP) \quad \text{and} \quad C(t) = C(t+kP), \quad \text{for all } t \text{ and } k.$$

Such systems are called linear periodically time-varying and are parameterized in state space by two matrix sequences

$$\big(A_1, \ldots, A_P\big) \quad \text{and} \quad \big(C_1, \ldots, C_P\big),$$

such that

$$A(t) = A_{(t-1) \bmod P+1} \quad \text{and} \quad C(t) = C_{(t-1) \bmod P+1}.$$

The nonuniqueness of the coefficients functions (A, C) of a periodic time-varying system's state space representation is given by (2). In order to preserve the periodicity of the coefficient functions, however, we restrict our attention to state transformations V to periodic, i.e., $V(t) = V_{(t-1) \bmod P+1}$, for some

$$\big(V_1, \ldots, V_P\big), \quad \text{where } V_i \in \mathbb{R}^{\mathtt{n}\times\mathtt{n}} \text{ and } \det(V_i) \neq 0.$$

The class of autonomous linear periodically time-varying systems with order at most $\mathtt{n}$ and period P is denoted by $\mathscr{L}_{0,\mathtt{n},P}$. (The zero subscript index stands for zero inputs.)

Problem 1 (*Realization of an Autonomous Linear Periodically Time-Varying System*)**.** Given a trajectory

$$y = \big(y(1), \ldots, y(T)\big),$$

of an autonomous linear periodically time-varying system $\mathscr{B}$, the period P of $\mathscr{B}$, and the state dimension $\mathtt{n}$ of $\mathscr{B}$, find a state space representation $\mathscr{B}(\widehat{A}, \widehat{C})$ of the system $\mathscr{B}$, i.e.,

$$\text{find } \widehat{\mathscr{B}} \in \mathscr{L}_{0,\mathtt{n},P} \quad \text{such that } y \in \widehat{\mathscr{B}}.$$

The assumption that the order $\mathtt{n}$ of $\mathscr{B}$ is given can be relaxed; see Note 2.

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