Automatica 50 (2014) 1641–1648

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Brief paper

# An event-triggered approach to state estimation with multiple point- and set-valued measurements[☆]

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ARTICLE INFO

*Article history:*
Received 19 April 2013
Received in revised form
30 November 2013
Accepted 10 March 2014
Available online 18 April 2014

*Keywords:*
Event-based estimation
Sensor fusion
Kalman filters
Wireless sensor networks

ABSTRACT

In this work, we consider state estimation based on the information from multiple sensors that provide their measurement updates according to separate event-triggering conditions. An optimal sensor fusion problem based on the hybrid measurement information (namely, point- and set-valued measurements) is formulated and explored. We show that under a commonly-accepted Gaussian assumption, the optimal estimator depends on the conditional mean and covariance of the measurement innovations, which applies to general event-triggering schemes. For the case that each channel of the sensors has its own event-triggering condition, closed-form representations are derived for the optimal estimate and the corresponding error covariance matrix, and it is proved that the exploration of the set-valued information provided by the event-triggering sets guarantees the improvement of estimation performance. The effectiveness of the proposed event-based estimator is demonstrated by extensive Monte Carlo simulation experiments for different categories of systems and comparative simulation with the classical Kalman filter.



## 1. Introduction

Event-based estimation strategy provides the possibility to maintain estimation performance under limited communication resources (Åström & Bernhardsson, 2002) and has attracted considerable attention in the control community for the past few years. For scalar linear systems, Imer and Basar (2005) and Rabi, Moustakides, and Baras (2006) studied the optimal event-based finite-horizon sensor transmission scheduling problems in continuous and discrete time, respectively. Li, Lemmon, and Wang (2010) extended the results to vector linear systems by relaxing the zero mean initial conditions and considering measurement noises. In Li and Lemmon (2011), the tradeoff between performance and average sampling period was analyzed, where a sub-optimal event-triggering scheme with a guaranteed least average sampling period was proposed. Rabi, Moustakides, and Baras (2012) considered the adaptive sampling for state estimation of linear continuous-time systems. In Wu, Jia, Johansson, and Shi (2013), the Minimum Mean Squared Error (MMSE) estimator was derived, and the tradeoff between communication rate and performance was analyzed. Shi, Chen, and Shi (2014) studied the likelihood estimation problem for a level-based event-triggering scheme, and the evaluation of upper and lower bounds on communication rates was discussed. Sijs and Lazar (2012) formulated a general description of event sampling, and a state estimator with a hybrid update was proposed to reduce the computational complexity.

The above results consider the scenario that only one event detector is used to process the measured state information from the sensor. There also exist many applications (e.g., in the context of wireless sensor networks) where multiple sensors with multiple event detectors are equipped to measure the state of the process. These invariably lead to sensor scheduling/fusion issues, which have been extensively studied for the case of periodic sampled systems (Alriksson & Rantzer, 2005; Mo, Ambrosinob, & Sinopoli, 2011; Shi & Chen, 2013). However, the effect of multiple event detectors on the MMSE estimates still remains unexplored, which is the basic motivation of our research. In this work, we consider the scenario that the process is measured by a network of sensors and that each sensor chooses to provide its latest measurement update according to its own event-triggering condition. In

☆ This work was supported by Natural Sciences and Engineering Research Council of Canada; in addition, the work by L. Shi was supported by an HK RGC GRF grant 618612. The material in this paper was not presented at any conference. This paper was recommended for publication in revised form by Associate Editor Martin Enqvist under the direction of Editor Torsten Söderström.

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http://dx.doi.org/10.1016/j.automatica.2014.04.004
0005-1098/

this case, the hybrid information is provided by the whole group of sensors as well as the event-triggering sets. For the sensors whose event-triggering conditions are satisfied, the exact values of the sensor outputs are known, providing "point-valued measurement information" to the estimator; for sensors that the event-triggering conditions are not satisfied, some information contained in the event-triggering sets is known to the estimator as well, to which we refer as "set-valued measurement information" in this paper. The basic goal is to find the MMSE estimate given the hybrid measurement information. As will be addressed later, the main issues arise from the computational aspect, due to the non-Gaussianity of the *a posteriori* distributions. Therefore we focus on the derivation of an approximate (due to the Gaussian assumption) MMSE estimate that possesses a simple structure but still inherits the important properties of the exact optimal estimate. In Sijs and Lazar (2012), a sum of Gaussians approach was utilized to solve the MMSE problem under a uniform distribution assumption; for the single-channel case, an alternative approach was proposed by Nguyen and Suh (2007), where an adaptive scheduling algorithm was developed to adjust the virtual moments of the measurement noises to achieve the improved estimation performance. The difference is that the aforementioned results would add an additional covariance matrix to the measurement noise covariance, while the present approach introduces a scalar weight when updating the estimation error covariance matrix (see Theorem 7). The main contributions are summarized as follows:

(1) An approximate MMSE estimate induced by the hybrid measurement information provided by a sequence of sensors has been derived. We show that the estimate is determined by the conditional mean and covariance of the innovations. The results are valid for general event-triggering schemes and reduce to the results obtained in Wu et al. (2013) if only one sensor and the level-based event-triggering conditions are considered.

(2) Insights on the optimal estimate when each sensor has only one channel are provided. In this case, closed-form recursive state estimate update equations are obtained. Utilizing the recent results on the partial order of uncertainty and information (Chen, 2011), we show that the exploration of the set-valued information guarantees the improved estimation performance in terms of smaller estimation error covariance. The results are equally applicable to multiple-channel sensors with uncorrelated/correlated measurement noises but separate event-triggering conditions on each channel.

(3) Extensive Monte Carlo experiments are performed to test the effectiveness of the proposed estimator. Compared with the Kalman filter that only exploits the received point-valued measurements, the proposed estimator provides almost-guaranteed improved performance, which is not sensitive to the sensor sequence used.

The rest of the paper is organized as follows: Section 2 presents the system description and problem setup. Section 3 presents the main results. Experimental verification using Monte Carlo simulation is provided in Section 4, followed by the concluding remarks in Section 5.

## 2. System description and problem setup

Consider a linear time-invariant process that evolves in discrete time driven by white noise:

$$x_{k+1} = Ax_k + w_k, \tag{1}$$

where $x_k \in \mathbb{R}^n$ is the state, and $w_k \in \mathbb{R}^n$ is the process noise, which is zero-mean Gaussian with covariance $Q \geq 0$. The initial value $x_0$ of the state is Gaussian with $\mathbf{E}(x_0) = \mu_0$, and covariance $P_0$. The state information is measured by a number of battery-powered sensors, which communicate with the state estimator through a wireless channel, and the output equations are

$$y_k^i = C^i x_k + v_k^i, \tag{2}$$

where $v_k^i \in \mathbb{R}^m$ is zero-mean Gaussian with covariance $R^i > 0$. In addition, $x_0$, $w_k$ and $v_k^i$ are uncorrelated with each other. We assume that the number of sensors equals $M$. Considering limitation in sensor battery capacity and the communication costs, an event-based data scheduler is equipped with each sensor $i$. At each time instant $k$, sensor $i$ produces a measurement $y_k^i$, and the scheduler of sensor $i$ tests the event-triggering condition

$$\gamma_k^i = \begin{cases} 0, & \text{if } y_k^i \in \Xi_k^i \\ 1, & \text{otherwise} \end{cases} \tag{3}$$

where $\Xi_k^i$ denotes the event-triggering set of sensor $i$ at time $k$ and decides whether to allow a data transmission. If $\gamma_k^i = 1$, sensor $i$ sends $y_k^i$ to the estimator through the wireless channel. Notice that the event-triggering scheme in (3) is fairly general and covers most schemes considered in the literature and industrial applications, e.g., the "send on delta" strategy and the level-based triggering conditions (not necessarily being symmetric). For many previously considered event-triggering schemes (e.g., the level-based event-triggering conditions in Shi et al. (2014) and Wu et al. (2013)), feedback communication from the estimator to the sensor is needed at certain time instants as the event is related to the innovation; however, since the event-triggering sets $\Xi_k^i$ can be designed offline, the remote estimator will have full knowledge of them without communication. In this way, the proposed results are applicable to battery-powered wireless sensor networks, where it is normally too costly to use feedback communication.

Since the main task is to study event-based estimation and sensor fusion, we assume that the capacity of the channel is greater than $M$ so that it is possible for the sensors to communicate with the estimator at the same time.

Let $\hat{x}_k^i$ denote the optimal estimate of $x_k$ after updating the measurement of the $i$th sensor and denote $P_k^i$ as the corresponding covariance matrix.[2] Denote $\mathbb{S}_+^n$ as the set of symmetric positive semidefinite matrices. Define the functions $h(\cdot)\colon \mathbb{S}_+^n \to \mathbb{S}_+^n$ and $\tilde{g}_i(\cdot,\cdot)\colon \mathbb{S}_+^n \times \mathbb{R} \to \mathbb{S}_+^n$ as follows:

$$\begin{aligned} h(X) &:= AXA^\top + Q, \\ \tilde{g}_i(X, \vartheta) &:= X - \vartheta X (C^i)^\top [C^i X (C^i)^\top + R^i]^{-1} C^i X. \end{aligned} \tag{4}$$

For brevity, we denote $\tilde{g}_i(X, 1)$ as $\tilde{g}_i(X)$. Denote $\mathcal{Y}_k := \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, \ldots, \mathcal{Y}_k^M\}$ as the collection of measurement information received by the estimator. Notice that if $\gamma_k^i = 1$, $\mathcal{Y}_k^i = \{y_k^i\}$; otherwise, $\mathcal{Y}_k^i = \{y_k^i | y_k^i \in \Xi_k^i\}$. In the latter case, although $y_k^i$ is unknown, it is still jointly Gaussian with $x_k$. Further define

$$\mathcal{I}_k^i := \{\mathcal{Y}_1, \mathcal{Y}_2, \ldots, \mathcal{Y}_{k-1}, \{\mathcal{Y}_k^1, \mathcal{Y}_k^2, \ldots, \mathcal{Y}_k^i\}\} \tag{5}$$

for $i \in \mathbb{N}_{1:M}$, and in this way, we are able to summarize all the information we have in $\mathcal{I}_k^i$ before considering the additional information $\mathcal{Y}_k^{i+1}$ from sensor $i+1$ at time $k$. The objective of our work is to explore the MMSE estimate of the process state (namely, $\mathbf{E}(x_k|\mathcal{I}_k^M)$) by taking into account all given information, namely, the set- and point-valued measurements provided by the sensor network as well as the event-triggering schemes.

When the state information is contained in combined point- and set-valued measurements, following a standard Bayesian

[2] Here we denote the 0th sensor as the case that no sensor information has been fused, namely, the prediction case.