Signal Processing 125 (2016) 216–220

Contents lists available at ScienceDirect

Signal Processing

journal homepage: www.elsevier.com/locate/sigpro

Fast communication

Bandpass phase shifter and analytic signal generator

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A R T I C L E I N F O

Article history:
Received 25 June 2015
Received in revised form
30 October 2015
Accepted 25 January 2016
Available online 5 February 2016

Keywords:
Signal processing
Digital filters
Communication theory
Analytic signal
Phase shifter
Hilbert transform

A B S T R A C T

In this note, a novel tunable bandpass filter/phase shifter implementation (with a Hilbert transformer as a special case) is proposed. The filter can also be used to synthesize a bandpass analytic signal from a real-valued signal. The novelty is the simple, yet elegant implementation that exploits the even and odd symmetry of the in-phase and quadrature carrier modulation.

1. Introduction

In some applications one needs to shift the phase of signal components by an angle θ without altering their amplitudes; a Hilbert transformer is an example. An ideal Hilbert transformer is an all-pass linear network with a phase response of $-\pi/2$ at positive frequencies and $\pi/2$ at negative frequencies. Therefore, all real sinusoidal signals get phase shifted by 90° when passed through a Hilbert transformer. In practice, Hilbert transformers are typically realized as filters or using Fourier transform on blocks of data in the following way: (1) a truncated or windowed version of the ideal Hilbert transform impulse response, $h(t)=1/(\pi t)$, (in MATLAB) is used as an impulse response to filter the input signal, or (2) given a block of N samples of a signal, its Hilbert transform is calculated using the fast Fourier transform (FFT) algorithm. However, in many practical applications one needs to compute the Hilbert transform of an indefinitely long sequence of signal samples at various spectral regions. Such applications include single sideband communication systems [4], defect diagnosis in rotating machinery [7], parametric coding of speech spectra [1,2], and using instantaneous frequencies to identify speakers [3]. For example, in a phase vocoder [2] Hilbert transform is typically computed in many bandpass regions of the signal spectrum. In this note, we propose a filter implementation which uses in-phase and quadrature modulation by a sinusoidal carrier to obtain a tunable bandpass filter and phase shifter, with Hilbert transformer as a special case. We also show how to use this phase shifter to build an analytic signal generator. The proposed method closely resembles Weaver's method of single sideband modulation [8].

2. Bandpass phase shifter

The filter structure shown in Fig. 1 is well known and widely used in communication systems. However, its use as a phase shifter/Hilbert transformer or analytic signal generator is not widely appreciated. The input signal $x(t)$ is assumed to be real-valued. It is typically a baseband signal (like speech) with a bandwidth of ω_b). The overall goal of the proposed method is to decompose $x(t)$ into N adjacent spectral regions spanning the frequency range from zero to ω_b using N bandpass filters. The output of these filters may then be converted to analytic signals, whose envelopes

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http://dx.doi.org/10.1016/j.sigpro.2016.01.015
0165-1684/

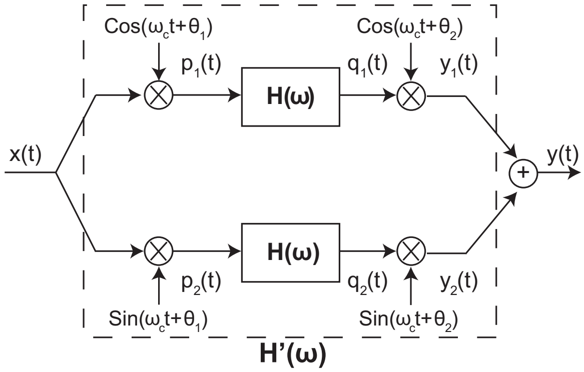

Fig. 1. A tunable bandpass filter (BPF). Frequency tuning is achieved by varying ω_c, while the phase of the output signals can be altered using the parameters θ_1 and θ_2. $H(\omega)$ could be any low pass filter, with a cutoff frequency ω_1 such that $\omega_1 < \omega_c$ and is a fraction of the input signal bandwidth ω_b.

and instantaneous frequencies can be used to represent the original signal $x(t)$. This is known as a Phase Vocoder [2] in speech processing.

Fig. 1 shows the structure of a bandpass filter centered at ω_c. The low-pass filter (LPF) sandwiched between the multipliers has a real-valued impulse response and its frequency response is denoted by $H(\omega)$. The LPF's cut off frequency, ω_1, is assumed to be a fraction of the original bandwidth, ω_b, of the signal $x(t)$, and is less than ω_c ($\omega_b > \omega_c > \omega_1$). The sine and cosine signals with frequency ω_c and phase θ_1 are used to complex demodulate the input signal to produce $q_1(t)$ and $q_2(t)$. After low pass filtering by $H(\omega)$ these signals are remodulated or upconverted by sine and cosine signals with the same frequency ω_c but with a different phase θ_2. To show the phase shifting properties of this structure, let us examine the frequency domain representations of the output $y(t)$ and the intermediate signals: $p_1(t)$, $p_2(t)$, $q_1(t)$ and $q_2(t)$ shown in Fig. 1. By inspection, we can write the following frequency domain representations of these signals:

$$P_1(\omega) = \tfrac{1}{2}\left[e^{j\theta_1}X(\omega-\omega_c)+e^{-j\theta_1}X(\omega+\omega_c)\right],$$
$$P_2(\omega) = \tfrac{1}{2j}\left[e^{j\theta_1}X(\omega-\omega_c)-e^{-j\theta_1}X(\omega+\omega_c)\right],$$
$$Q_1(\omega) = \tfrac{1}{2}\left[e^{j\theta_1}X(\omega-\omega_c)+e^{-j\theta_1}X(\omega+\omega_c)\right]H(\omega),$$
$$Q_2(\omega) = \tfrac{1}{2j}\left[e^{j\theta_1}X(\omega-\omega_c)-e^{-j\theta_1}X(\omega+\omega_c)\right]H(\omega).$$

The cut off frequency ω_1 of the LPF $H(\omega)$ is such that $\omega_c > \omega_1$. The Fourier transforms of $y_1(t)$ and $y_2(t)$ are as follows:

$$\begin{aligned}Y_1(\omega) = \tfrac{1}{4}\Big[&e^{j(\theta_2+\theta_1)}X(\omega-2\omega_c)H(\omega-\omega_c)\\ &+e^{j(\theta_2-\theta_1)}X(\omega)H(\omega-\omega_c)+e^{j(-\theta_2+\theta_1)}X(\omega)H(\omega+\omega_c)\\ &+e^{-j(\theta_2+\theta_1)}X(\omega+2\omega_c)H(\omega+\omega_c)\Big],\\ Y_2(\omega) = -\tfrac{1}{4}\Big[&e^{j(\theta_2+\theta_1)}X(\omega-2\omega_c)H(\omega-\omega_c)\\ &-e^{j(\theta_2-\theta_1)}X(\omega)H(\omega-\omega_c)-e^{j(-\theta_2+\theta_1)}X(\omega)H(\omega+\omega_c)\\ &+e^{-j(\theta_2+\theta_1)}X(\omega+2\omega_c)H(\omega+\omega_c)\Big].\end{aligned} \tag{1}$$

The spectrum of the output signal, $Y(\omega) = Y_1(\omega)+Y_2(\omega)$, simplifies to the following:

$$Y(\omega) = \tfrac{1}{2}\left[e^{j\Delta\theta}H(\omega-\omega_c)+e^{-j\Delta\theta}H(\omega+\omega_c)\right]X(\omega), \tag{2}$$

where $\Delta\theta = \theta_2-\theta_1$. Notice that the terms containing $X(\omega\pm 2\omega_c)$ in $Y_1(\omega)$ and $Y_2(\omega)$ cancel each other independent of the phase values θ_1 and θ_2. Therefore the overall frequency response of the entire filter in Fig. 1, denoted by $H'(\omega) = Y(\omega)/X(\omega)$, is given by the following expression:

$$H'(\omega) = \tfrac{1}{2}\left[e^{j\Delta\theta}H(\omega-\omega_c)+e^{-j\Delta\theta}H(\omega+\omega_c)\right]. \tag{3}$$

Clearly, $H'(\omega)$ is a bandpass filter with frequency response centered at ω_c with bandwidth $2\omega_1$. Bandpass filter realizations as in Fig. 1 with $\theta_1=\theta_2=0$ can be found in text books (e.g refer to [5, p. 176]). The purpose of this note is to point out that the filter structure in Fig. 1 can be used to rotate the phase of signal components within the pass band of $H'(\omega)$ by any desired angle $\Delta\theta$. Hence it can also be used as a bandpass phase shifter, especially for discrete-time implementations. Further, if we choose $\Delta\theta$ to be $\pi/2$, it becomes a "bandpass Hilbert transformer." This can also be used to generate an analytic signal as shown in the next section.

Let us consider an example to examine the output and intermediate signals for a specific input $x(t) = A\cos(\omega_0 t)$. Let ω_0 be such that $|\omega_c-\omega_0| < \omega_1$ (where $H'(\omega)$ is centered at ω_c and ω_1 is the cut off frequency of $H(\omega)$ shown in Fig. 1). That is, the tone $x(t)$ lies in the passband of the filter $H'(\omega)$. Then

$$\begin{aligned}p_1(t) &= A\cos(\omega_0 t)\cos(\omega_c t+\theta_1)\\ &= \frac{A}{2}[\cos(\omega_0 t+\omega_c t+\theta_1)+\cos(\omega_0 t-\omega_c t-\theta_1)],\\ p_2(t) &= A\cos(\omega_0 t)\sin(\omega_c t+\theta_1)\\ &= \frac{A}{2}[\sin(\omega_0 t+\omega_c t+\theta_1)-\sin(\omega_0 t-\omega_c t-\theta_1)].\end{aligned} \tag{4}$$

Assuming an ideal LPF, $H(\omega)$, with cutoff frequency $\omega_1 < \omega_c$, the LPF outputs are $q_1(t) = \frac{A}{2}[\cos(\omega_0 t-\omega_c t-\theta_1)]$ and $q_2(t) = \frac{A}{2}[-\sin(\omega_0 t-\omega_c t-\theta_1)]$. Then the signals $y_1(t)$ and $y_2(t)$ are

$$\begin{aligned}y_1(t) &= \frac{A}{2}[\cos(\omega_0 t-\omega_c t-\theta_1)\cos(\omega_c t+\theta_2)]\\ &= \frac{A}{4}[\cos(\omega_0 t-\theta_1+\theta_2)+\cos(\omega_0 t-2\omega_c t-\theta_1-\theta_2)],\\ y_2(t) &= \frac{A}{2}[-\sin(\omega_0 t-\omega_c t-\theta_1)\sin(\omega_c t+\theta_2)]\\ &= \frac{A}{4}[\cos(\omega_0 t-\theta_1+\theta_2)-\cos(\omega_0 t-2\omega_c t-\theta_1-\theta_2)].\end{aligned} \tag{5}$$

Therefore the output $y(t)$ is

$$y(t) = y_1(t)+y_2(t) = \frac{A}{2}\cos(\omega_0 t-\theta_1+\theta_2). \tag{6}$$

Note that $\cos(\omega_0 t-2\omega_c t-\theta_1-\theta_2)$ terms cancel irrespective of θ_1 and θ_2. Additionally the phase of $x(t)$ is rotated by $\Delta\theta(=\theta_2-\theta_1)$.

2.1. Analytic signal generator

Consider the two bandpass filter structures shown in Fig. 2a and b that are derived from Fig. 1. We refer to them as "Cos–Cos" and "Cos–Sin" filters respectively. We coined these