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Brief paper

Source localization by gradient estimation based on Poisson integral[☆]

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ABSTRACT

We consider the problem of localizing the source of a diffusion process. The source is supposed to be isotropic, and several sensors, equipped on a vehicle moving without position information, provide pointwise measures of the quantity being emitted. The solution we propose is based on computing the gradient – and higher-order derivatives such as the Hessian – from Poisson integrals: in opposition to other solutions previously proposed, this computation does neither require specific knowledge of the solution of the diffusion process, nor the use of probing signals, but only exploits properties of the PDE describing the diffusion process. The theoretical results are illustrated by simulations.

1. Introduction

The problem of source localization consists in finding, by one or several mobile or fixed sensors arrays, possibly cooperating with each other, the point or the spatial region from which a quantity of interest is being emitted. Source-seeking agents can be fixed sensors, that collect and exchange some information about the signal field and try to identify the position of the source (or the smallest region in which it is included), or moving devices equipped with one or more sensors, that physically reach the source in an individual or cooperative way.

This research area is attracting rapidly increasing interest, in particular in applications where the agents have limited or no position information; for instance, source localization is relevant to many applications of vapor emitting sources (Porat & Nehorai, 1996), such as explosive detection, drug detection, sensing leaking or hazardous chemicals, pollution sensing and environmental studies. Sound source localization (Zhang, Florêncio, Ba, & Zhang, 2008) is pertinent for intelligent conference call systems that identify the speakers to improve sound and video quality. Other applications also include heat source localization, vent sources in underwater field, and medical applications to explore internal brain activity by using surface sensors.

1.1. Overview of source-seeking

A variety of methods exists in the literature to treat the problem of source localization and related issues. Many techniques deal with formulations associated with isotropic diffusion processes, and several identification methods have been devised to estimate the source position (Matthes, Gröll, & Keller, 2004; Porat & Nehorai, 1996); more fundamental problems, such as source identifiability and optimal sensor placement, are discussed in depth in Khapalov (2010). This approach, that can be viewed as an inverse problem formulation for partial differential equations, has the drawbacks of a heavy computation, and the requirement of the explicit knowledge of the closed-form solution of the PDE describing the diffusion process.

A different line of research consists in reconstructing an approximation of the gradient field of the measured quantity, and moving towards the source along the gradient direction; this can be done either directly, via a method developed for the particular problem at hand, or implicitly, by estimating the gradient via different techniques. The first contribution making use of an explicit *ad hoc* gradient computation can be found in Burian, Yoerger, Bradley, and Singh (1996), where the agent obtains different measurements of a hydrothermal plume and performs a least-square gradient estimation; in Baronov and Baillieul (2008) unicycle vehicles are driven towards a source by a control law

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related to the geometry of the diffusion process. The gradient is estimated in an implicit way in Wu, Couzin, and Zhang (2012), where the authors take inspiration from fish swarms to design an algorithm adapted also for seeking the source of a turbulent flow (Wu, Chang, & Zhang, 2013).

In contrast to the previously mentioned methods, the so-called "extremum seeking" methodology is not based on any particular structure or knowledge of the diffusion solution; the method only applies for moving sensors, as it relies on the idea of collecting rich enough information to approximate the gradient through the use of a periodic probing signal or an oscillatory motion. Extremum-seeking has been applied in a wide range of engineering applications; adaptations of this idea to the problem of source localization are presented in Ghods (2011) and references therein, where the control of the nonholonomic unicycle is performed first on the forward velocity, then on angular one, while in the last case the authors combine the two strategies in an elegant way. Extremum-seeking has been used also in some contributions on 3D source localization (Cochran, Ghods, & Krstić, 2008; Cochran, Siranosian, Ghods, & Krstić, 2009).

Another technique worth mentioning is given by stochastic source-localization. Stochastic methods are based on a function that describes the probability rate of a change of direction, and often mimic biological behaviors observed for example on fish swarms or in bacteria movements. A contribution in this direction is the *Optimotaxis* (Mesquita, Hespanha, & AAström, 2008), where the vehicles move with a random motion similar to *Escherichia coli*'s "run and tumble"; this method can be also used with nonquadratic-like signal profiles, including the ones with multiple maxima. Another contribution can be found in Menon and Ghose (2012), where the authors localize a source of polluting substance and also track the boundary of the contaminated region.

All the methods discussed above have been also used to develop a distributed approach to the source-localization problem. One of the first in this direction is (Moreau, Bachmayer, & Leonard, 2003), where it is assumed that each vehicle, modeled with simple integrator dynamics, can measure the full gradient, and the authors develop a twofold algorithm with a gradient-descent term and inter-vehicle forcing terms. Extremum-seeking is applied in a collaborative manner in Ghods and Krstić (2010), in a 1-dimensional framework. Two distributed stochastic source localizations are in Sahyoun, Djouadi, and Qi (2010), where a group of chemical sensors takes measures of a plume concentration values to estimate the source position via a stochastic approximation technique, and in Rabbat and Nowak (2004), in which the authors use the sensor measurements to estimate the model parameters of the concentration plume. A collaborative control law to steer a fleet of AUVs (autonomous underwater vehicles) to the source of a signal distribution using only direct signal measurements by a circular formation of agents is presented in Briñón Arranz, Seuret, and Canudas de Wit (2011) and Moore and Canudas de Wit (2010); these ideas are formalized and extended in Canudas de Wit, Garin, Fabbiano, Rouchon, and Rousseau (2012). Some recent works using a distributed approach deal in particular with acoustic source localization; an example is (Yong, Qing-Hao, Yuxiu, & Ming, 2012), in which the authors solve the so-called "energy-based source localization" problem, i.e., detecting the presence of a source emitting an acoustic signal that attenuates in space by a field of sensors able to measure the signal's energy, by proposing a new optimization method called "projection onto the nearest local minimum".

1.2. Main paper contribution

We present here a new method, suitable for a single-vehicle n-dimensional source localization, based on a direct gradient computation. This technique does not require specific knowledge of the solution of the diffusion process, but only exploits properties of the PDE that generates the diffusion process, and can compute the gradient direction from the pointwise concentration samples collected by multiple sensors arranged on a spherical surface, with a small computation load; it does not make use of a probing signal either. We note also that the vehicle does not need any position information, since the heading references can be computed with respect to the vehicle's orientation in its local frame.

The gradient computation, necessary to perform the source search, is based on the Poisson integral formula; this approach allows also for higher-order derivatives computation (e.g., the Hessian), which can be useful to implement different control laws. Moreover, this is intrinsically high-frequency filtering, since derivatives are computed using integrals, and it makes the method less sensitive to measurement noise.

The approach is based on the assumption, justified for isotropic diffusive sources in steady-state, that the diffusion process is described by the Laplacian PDE. The paper formalizes and extends previous ideas from Briñón Arranz et al. (2011) and Moore and Canudas de Wit (2010), where the gradient has been approximated by the sum of pointwise measurements around a circle weighted by the position vector of each sampler with respect to its center of rotation.

2. Problem formulation

Before starting we fix here some notation we will use throughout the paper. A point in an n-dimensional space is represented by the vector $\boldsymbol{x} = [x_1\ x_2\ \cdots\ x_n]^{\mathrm{T}}$; ∇f is the gradient of the function f, and $\nabla^2 = \sum_{i=1}^{n} \partial^2/\partial x_i^2$ is the Laplacian operator. For an open set Ω, $\partial\Omega$ denotes its border, and $\bar{\Omega} = \Omega \cup \partial\Omega$ its closure. $B_r(\boldsymbol{c})$ indicates the ball of radius r centered in $\boldsymbol{c}$; the area of the surface of $B_r(\boldsymbol{c}) \subset \mathbb{R}^n$ is given by $\omega_n r^{n-1}$, ω_n being the area of the corresponding unit sphere, and $\mathrm{d}S_{B_r(\boldsymbol{c})}$ denotes the infinitesimal element of surface of $B_r(\boldsymbol{c})$. Integrals of vector- or matrix-valued quantities are intended as entry-wise integrals. Finally, $[a]$ denotes the rounding of a, i.e., the integer closest to a.

We consider steady-state behaviors of homogeneous diffusion processes caused by an isotropic source emitting at a constant rate. Such a process is governed by the well-known diffusion equation

$$\frac{\partial f(\boldsymbol{x}, t)}{\partial t} - k\nabla^2 f(\boldsymbol{x}, t) = 0, \quad \forall \boldsymbol{x} \in \Omega,\ t \geq 0, \tag{1}$$

where f is the concentration variable, k is a diffusion coefficient, and $\Omega \subset \mathbb{R}^n$ (see Folland, 1995). In particular, as depicted in Fig. 1(a), we consider the region of interest $\tilde{\Omega} \subset \mathbb{R}^n$ as a connected bounded set $\tilde{\Omega} = \Omega \cup \Omega_s$, where Ω_s is the connected bounded set that identifies the source, and therefore we have that $\partial\Omega = \partial\tilde{\Omega} \cup \partial\Omega_s$. The values of f on the inner boundary $\partial\Omega_s$ are imposed by the source, so we can assume that values of f on $\partial\Omega_s$ are higher than the ones on $\partial\tilde{\Omega}$. As we will see below, we have that $\max f(\boldsymbol{x}) \in \partial\Omega$, that means, for our previous consideration, that it lies on $\partial\Omega_s$; our source localization problem is then mathematically equivalent to the problem of finding the maximum of f.

Once the steady-state has been reached, supposing that the source is still emitting at a constant rate (this happens in many cases of practical interest, e.g., in a heating process, or in the dispersion of a chemical substance), or that possible source variations are slow in the time-scale of interest, the diffusion equation (1) reduces to the Laplace equation

$$\nabla^2 f(\boldsymbol{x}) = 0, \quad \forall \boldsymbol{x} \in \Omega, \tag{2}$$

whose solutions are called "harmonic". These functions have many properties; among them, they satisfy the *maximum principle*, which