Automatica 50 (2014) 1730–1734

Contents lists available at ScienceDirect

Automatica

journal homepage: www.elsevier.com/locate/automatica

Technical communique

A study of disturbance observers with unknown relative degree of the plant[☆]

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ARTICLE INFO

Article history:
Received 22 August 2013
Received in revised form
5 January 2014
Accepted 3 April 2014
Available online 15 May 2014

Keywords:
Robust stability
Disturbance observer
Relative degree
Newton diagram

ABSTRACT

Robust stability of the disturbance observer (DOB) control system is studied when the relative degree of the plant is not the same as that of the nominal model. The study reveals that the closed-loop system can easily become unstable with sufficiently fast Q-filter when the relative degree of the plant is not known. In a few cases of unknown relative degree, however, robust stability can be obtained, and we present a design guideline of the nominal model, as well as the Q-filter, for that purpose. Moreover, a universal design of DOB is given for a plant whose relative degree is uncertain but less than or equal to four.

1. Introduction

The disturbance observer (DOB) based controller has been widely used among control engineers since it has a powerful ability of uncertainty compensation and disturbance attenuation. (See, e.g., Kempf & Kobayashi, 1999, Kobayashi, Katsura, & Ohnishi, 2007, Lee & Tomizuka, 1996, Wang & Tomizuka, 2004 and Yi, Chang, & Shen, 2009 and references therein.) The standard DOB control system is illustrated in Fig. 1. In the figure, $P(s)$ and $P_n(s)$ represent the transfer functions of the uncertain plant and its nominal model, and signals d and r represent the input disturbance and the reference, respectively. It is assumed that $P(s) \in \mathcal{P}$ where $\mathcal{P}$ is a known set of uncertain plants. The controller $C(s)$ is designed for the nominal model $P_n(s)$. The Q-filter $Q(s)$ is a *stable* low-pass filter, which usually has the form (Choi, Yang, Chung, Kim, & Suh, 2003; Lee & Tomizuka, 1996; Shim & Jo, 2009) of

$$Q(s) = \frac{b_k(\tau s)^k + b_{k-1}(\tau s)^{k-1} + \cdots + b_0}{(\tau s)^l + a_{l-1}(\tau s)^{l-1} + \cdots + a_1(\tau s) + a_0} \quad (1)$$

where $\tau > 0$ is the filter time constant, k and l are nonnegative integers with $b_k \neq 0$. We assume $a_0 = b_0$ and $l - k \geq \mathrm{r.deg}(P_n)$, where $\mathrm{r.deg}(P_n)$ stands for relative degree of P_n.

The output y is represented as $y(s) = T_{yr}(s)r(s) + T_{yd}(s)d(s)$ where T_{yr} is the transfer function from r to y and so on. For sufficiently small $\tau > 0$, it can be shown that $T_{yr}(j\omega) \approx P_nC/(1 + P_nC)(j\omega)$ and $T_{yd}(j\omega) \approx 0$ on a finite frequency range, which implies the recovery of the nominal closed-loop steady-state performance. (See, e.g., Shim & Jo, 2009 and Shim & Joo, 2007 for more details.) This property holds only when all the transfer functions are stable. Therefore, the question of interest is the robust stability of the closed-loop system in Fig. 1 under the variation of $P(s) \in \mathcal{P}$, which depends on the selection of $Q(s)$ and $P_n(s)$. This question has been studied under the perspective of small-gain theorem in Choi et al. (2003), Kim and Chung (2003), Kong and Tomizuka (2013) and Schrijver and van Dijk (2002), where only a sufficient stability condition is presented that is conservative in nature. On the other hand, some necessary and sufficient condition for robust stability is presented in Shim and Jo (2009) and Shim and Joo (2007) under the assumption that the time constant of the Q-filter is sufficiently small, which has played a key role in extending to nonlinear systems (Back & Shim, 2008) and embedding an internal

☆ This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MEST) (No. 2011-0010233). The material in this paper was partially presented at the 11th International Conference on Control, Automation and Systems (ICCAS 2011), October 26–29, 2011, Gyeonggi-do, Korea, and the 51st IEEE Conference on Decision and Control (CDC 2012), December 10–13, 2012, Maui, Hawaii, USA. This paper was recommended for publication in revised form by Associate Editor Constantino M. Lagoa under the direction of Editor André L. Tits.

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http://dx.doi.org/10.1016/j.automatica.2014.04.015
0005-1098/

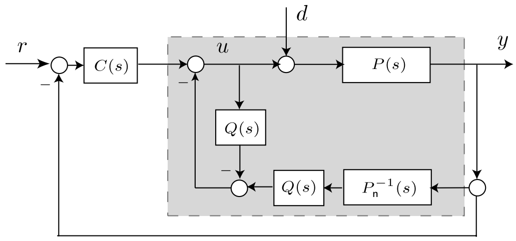

Fig. 1. Structure of the DOB control system. The shaded region represents the real plant $P(s)$ augmented with the DOB.

model into the DOB structure (Park, Joo, Shim, & Back, 2012). However, the study of Back and Shim (2008), Park et al. (2012), Shim and Jo (2009) and Shim and Joo (2007) assumes that the relative degree of plant is the same as that of nominal model.

In this paper,[2] we study the robust stability of the DOB-based control system (Fig. 1) when the relative degree of plant is not exactly known and so it happens to be different from that of nominal model. This case often occurs in real world control applications. For instance, $\mathrm{r.deg}(P) > \mathrm{r.deg}(P_n)$ when the actuator dynamics is ignored, or when there is unmodeled dynamics for the plant. Inspired by the fact that the characteristic equation for stability is of the form that appears in the 'higher-order root locus technique' (Hahn, 1981), a condition for robust stability is derived by utilizing the Newton diagram. The derived condition reveals a few facts such as: (1) if $\mathrm{r.deg}(P) = \mathrm{r.deg}(P_n) + 1$, robust stability can be achieved by an appropriate design of P_n as well as Q. (2) If $1 \le \mathrm{r.deg}(P) \le 2$, then robust stability is always achievable. (3) If $\mathrm{r.deg}(P) \ge \mathrm{r.deg}(P_n) + 2$ or $\mathrm{r.deg}(P_n) > \mathrm{r.deg}(P) > 2$, then robust stabilization is not possible with sufficiently small τ no matter how P_n, C, and Q are selected. A universal design of DOB is also discussed for the special case where $\mathrm{r.deg}(P)$ is unknown but $1 \le \mathrm{r.deg}(P) \le 4$.

Notation. Let $D(s)$ be a polynomial with real coefficients expressed as $D(s) = d_n s^n + d_{n-1}s^{n-1} + \cdots + d_1 s + d_0$. The polynomial $D(s)$ is said to be of *degree n* if $d_n \neq 0$, which will be denoted by $\deg(D) = n$. For a transfer function $G(s) = N(s)/D(s)$ (it is assumed that $N(s)$ and $D(s)$ are coprime polynomials), the degree and the relative degree of $G(s)$ are defined as $\deg(D)$ and $\deg(D) - \deg(N)$, respectively, and the latter will be denoted by $\mathrm{r.deg}(G)$. The high-frequency gain of $G(s)$ is defined as $\lim_{s\to\infty} s^{\mathrm{r.deg}(G)}G(s)$ and denoted by $\kappa(G)$. Finally, LHP (RHP, respectively) stands for the *open* left (right, respectively) half plane.

2. Robust stability

We assume that $P(s)$ and $P_n(s)$ are strictly proper while $C(s)$ is proper. Let P, P_n, C, and Q in Fig. 1 be represented by the ratios of coprime polynomials, that is, $P(s) = N(s)/D(s)$, $P_n(s) = N_n(s)/D_n(s)$, $C(s) = N_c(s)/D_c(s)$, and $Q(s) = N_Q(s;\tau)/D_Q(s;\tau)$ (in which, the dependence of N_Q and D_Q on τ is explicitly indicated). Moreover, we assume that there is no unstable pole-zero cancellation in $P_n(s)C(s)$ and in $P_n^{-1}(s)Q(s)$. Then, it can be shown that, for given $\tau > 0$, the closed-loop system is internally stable if and only if the characteristic polynomial

$$\delta(s;\tau) := (DD_c + NN_c)N_nD_Q + N_QD_c(ND_n - N_nD)$$

[2] Preliminary versions of this paper have been presented at *Int. Conf. on Control, Automation and Systems, 2011*, where the Q-filter is just a first order system and the relative degree of P_n is limited to one, and at *51st IEEE Conf. on Dec. and Control, 2012*, where the case $\mathrm{r.deg}(P) < \mathrm{r.deg}(P_n)$ is not considered.

is Hurwitz. Define

$$p_\alpha(s) := N(N_cN_n + D_cD_n)$$
$$p_\beta(s) := N_n(N_cN + D_cD) \tag{2}$$

and let $m_\alpha := \deg(ND_cD_n)$, $m_\beta := \deg(N_nD_cD)$, and α_i, β_i be such that

$$p_\alpha(s) = \alpha_{m_\alpha}s^{m_\alpha} + \alpha_{m_\alpha-1}s^{m_\alpha-1} + \cdots + \alpha_0$$
$$p_\beta(s) = \beta_{m_\beta}s^{m_\beta} + \beta_{m_\beta-1}s^{m_\beta-1} + \cdots + \beta_0.$$

It should be kept in mind that $m_\beta - m_\alpha = \mathrm{r.deg}(P) - \mathrm{r.deg}(P_n)$, and that $\beta_{m_\beta}/\alpha_{m_\alpha} = \kappa(P_n)/\kappa(P)$. Let $\bar{k}$ $(\le k)$ be such that $a_0 = b_0, \ldots, a_{\bar{k}} = b_{\bar{k}}$, and $a_{\bar{k}+1} \neq b_{\bar{k}+1}$ in $Q(s)$. Then, it follows that (with $a_l = 1$ for convenience)

$$\begin{aligned}\delta(s;\tau) &= p_\beta(s)D_Q(s;\tau) + (p_\alpha(s) - p_\beta(s))N_Q(s;\tau)\\ &= p_\beta(s)\sum_{i=0}^{l}a_i(\tau s)^i + (p_\alpha(s) - p_\beta(s))\sum_{i=0}^{k}b_i(\tau s)^i\\ &= \sum_{i=0}^{\bar{k}}(\tau s)^i a_i p_\alpha(s) + \sum_{i=\bar{k}+1}^{k}(\tau s)^i\big(a_ip_\beta(s)\\ &\quad + b_i(p_\alpha(s) - p_\beta(s))\big) + \sum_{i=k+1}^{l}(\tau s)^i a_i p_\beta(s).\end{aligned} \tag{3}$$

Note that $\deg(\delta(s;\tau)) = l + m_\beta$ if $\tau > 0$, and the locations of $l + m_\beta$ roots, when τ is sufficiently small, are of interest. Since $\delta(s;0) = a_0p_\alpha(s)$ and $\deg(\delta(s;0)) = m_\alpha$, it is clear that m_α roots out of $l+m_\beta$ roots of $\delta(s;\tau)$ converge to the roots of $p_\alpha(s)$ as $\tau \to 0$, while the remaining $l + m_\beta - m_\alpha$ roots tend to infinity (see Shim & Jo, 2009 for more rigorous arguments).

Here we recall the result of Shim and Jo (2009), with the set $\mathcal{P}$ being a collection of transfer functions whose coefficients belong to certain (known) bounded intervals.

Proposition 1 (*Shim & Jo, 2009*)**.** *Suppose that* $\mathrm{r.deg}(P) = \mathrm{r.deg}(P_n)$ *for all* $P(s) \in \mathcal{P}$. *Then, there exists a constant* $\tau^* > 0$ *such that, for all* $0 < \tau \le \tau^*$, *the closed-loop system is robustly stable if all the following conditions hold:*

(H1) *all* $P(s) \in \mathcal{P}$ *are of minimum phase,*
(H2) *the transfer function* $P_nC/(1 + P_nC)$ *is stable,*
(H3) *the polynomial*

$$p_f(s) := D_Q(s;1) + \left(\lim_{s\to\infty}\frac{P(s)}{P_n(s)} - 1\right)N_Q(s;1)$$

is Hurwitz for all $P(s) \in \mathcal{P}$.

On the contrary, for given $P \in \mathcal{P}$, *there is* $\tau^* > 0$ *such that, for all* $0 < \tau \le \tau^*$, *the closed-loop system is unstable if* $P_nC/(1 + P_nC)$ *has some poles in the RHP, or some zeros of* $P(s)$ *or some roots of* $p_f(s) = 0$ *are located in the RHP.*

Remark 2. It is observed that the conditions (H1) and (H2) are equivalent to $p_\alpha(s)$ being Hurwitz (see (2)), so that m_α roots of $\delta(s;\tau)$ have negative real parts for sufficiently small τ. On the other hand, the condition (H3) constrains the other $l+m_\beta-m_\alpha = l$ (since $m_\beta = m_\alpha$ if $\mathrm{r.deg}(P) = \mathrm{r.deg}(P_n)$) roots to remain in the LHP.

Note that Proposition 1 is not conclusive when any one of conditions is marginal (e.g., if some roots of $p_f(s)$ are located on the imaginary axis), by which the condition is 'almost' necessary and sufficient. It is inconclusive particularly when $\mathrm{r.deg}(P) > \mathrm{r.deg}(P_n)$ because $\lim_{s\to\infty} P(s)/P_n(s) = 0$ so that $p_f(s)$ has at least one root at the origin of complex plane (recall that $a_0 = b_0$). The polynomial $p_f(s)$ is not even defined when $\mathrm{r.deg}(P) < \mathrm{r.deg}(P_n)$.