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Passivity-based control for hybrid systems with applications to mechanical systems exhibiting impacts[☆]

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ABSTRACT

Motivated by applications of systems interacting with their environments, we study the design of passivity-based controllers for a class of hybrid systems in which the energy dissipation may only happen along either the continuous or the discrete dynamics. A general definition of passivity, encompassing the said special cases, is introduced and, along with detectability and solution conditions, linked to stability and asymptotic stability of compact sets. The proposed results allow us to take advantage of the passivity property of the system at flows or at jumps and are employed to design passivity-based controllers for the class of hybrid systems of interest. Two applications, one pertaining to a point mass physically interacting with a wall and another about controlling a ball bouncing on an actuated surface, illustrate the definitions and results throughout the paper.

1. Introduction

1.1. Background

Dissipativity and its special case, passivity, provide a useful physical interpretation to stability and stabilizability problems as they establish a relationship between the energy injected in and dissipated by a system. Several textbooks (Isidori, 1998; Khalil, 1996; Sepulchre, Jankovic, & Kokotovic, 1997; van der Schaft, 2000) and seminal papers (Kokotovic & Sussman, 1989; Lin & Byrnes, 1995; Ortega, van der Schaft, Mareels, & Maschke, 2001; Willems, 1972) document dissipativity and passivity concepts, sufficient conditions linking to stability, and passivity-based feedback control designs; for a detailed survey on the latter see Ortega and Garcia-Canseco (2004). For passive systems, the passivity-based control design technique has been shown to be particularly useful in designing controllers that can be well understood from an energetic perspective. The problem of stabilizing a system to a given equilibrium point, in particular, is addressed by designing a feedback controller such that the overall energy function has the desired form and minimum. With such a function, convergence is obtained by selecting the input so that the energy of the system is dissipated. Modifications of the energy function and of the dissipation rate are often referred to as *energy shaping* and *damping injection* respectively (see, e.g., Ortega et al., 2001).

Dissipativity and passivity have been recently considered for several types of hybrid systems. Passivity of switching systems was investigated in Pogromsky, Jirstrand, and Spangeous (1998). Motivated by haptic and teleoperation applications, a notion of passivity for systems in which the controller switches between different operative modes was proposed in Zefran, Bullo, and Stein (2001). Results about dissipativity of switching systems appeared also in Zhao and Hill (2008), where multiple storage functions were considered. Passivity and passivity-based control for systems undertaking impacts and unilateral constraints have been investigated in Brogliato, Lozano, and Egeland (2007) by first extending the Lagrange–Dirichlet theorem to a class of nonsmooth Lagrangian systems. The results therein are applied to mechanical systems including robotic manipulators with rigid and flexible joints.

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Controllability and stabilizability issues for nonsmooth mechanical systems have been also considered in Brogliato (2003) for a class of complementarity systems (for more details regarding such a class of systems the reader is also referred to Brogliato (1996)). For such systems, in Morărescu and Brogliato (2010a), passivity-based controllers are proposed, while, in Morărescu and Brogliato (2010b), tracking control problems are considered. In Leine and van de Wouw (2007), the stability of multiple degree-of-freedom mechanical systems subject to frictional unilateral/bilateral constraints is investigated and the attractivity of equilibria is shown to be linked to dissipativity properties. In Spong, Holm, and Lee (2007), passivity-based control techniques are employed to regulate walking for a class of bipedal robots (see also Westervelt, Grizzle, Chevallereau, Choi, & Morris, 2007). In this work, impact Poincaré maps are considered as a tool to investigate stability of periodic orbits characterizing the desired walking behavior. In Haddad, Chellaboina, and Nersesov (2006), the authors consider dissipativity theory for a class of impulsive dynamical systems. In particular, the proposed framework considers different inputs and outputs maps for respectively the continuous-time evolution and the instantaneous changes, and results linking observability to asymptotic stability for the design of feedback controllers are presented. Moreover, in Haddad and Hui (2008), the authors also present energy-based hybrid controllers for impulsive dynamical systems. More recently, a general notion of dissipativity for a class of hybrid systems was linked to detectability and used to establish asymptotic stability for large-scale interconnections of hybrid systems in Teel (2010).

1.2. Contributions

Building from the ideas in Haddad et al. (2006) and Teel (2010), and driven by applications of mechanical systems interacting with their environment, this paper studies the design of passivity-based controllers for a class of hybrid systems. In particular, we study the case of hybrid systems in which the energy dissipation may only happen along either the continuous or the discrete dynamics. For such systems, two weak notions of passivity, respectively *flow-passivity*, in which dissipation happens along flows, and *jump-passivity*, in which dissipation happens along jumps, as well as their strict and output versions are introduced and linked to asymptotic stability.

More precisely, in Section 3.2, we introduce first general definitions of passivity, strict passivity, and output strict passivity for hybrid systems. Inspired by Haddad et al. (2006), the proposed definitions consider different input and output maps for the continuous and the discrete dynamics, respectively, and encompass also the two hybrid specific cases of flow- and jump-passivity. Then, with the passivity definitions at hand, in Section 3.4 we establish basic properties of passive hybrid systems. In particular, we show that passivity and strict passivity with respect to a compact set imply, respectively, 0-input stability and 0-input asymptotic stability. Furthermore, we also show that output strict passivity with respect to a compact set implies 0-input asymptotic stability provided that a detectability property holds true. These general results are then specialized to the cases of flow- and jump-passivity, showing how the hybrid specific notions of passivity can be linked to asymptotic stability under weaker conditions than when using the standard notions. In particular, for the output strict passivity cases, it is shown that 0-input asymptotic stability holds under a different detectability property and additional conditions on solutions.

The established basic properties are then employed for the design of passivity-based controllers in Section 4. In particular, for the hybrid-specific notion of flow-passivity, we establish that a static output–feedback law for the flow input asymptotically stabilizes a compact set when the resulting closed-loop system has a detectability property and jumps in the solutions are separated by a (uniformly) nonzero amount of flow time. A similar result holds also for the jump-passivity case, for which we establish that static output–feedback for the jump input asymptotically stabilizes a compact set provided that solutions to the resulting closed-loop system, besides satisfying a detectability property, are Zeno.

We exercise the results in two applications. The first one consists of a mechanical system capturing the dynamics of a simple robotic manipulator that is required to interact physically with the environment through the effect of a control input affecting the continuous dynamics (see also Brogliato, 1996, Section 7.3, Brogliato et al., 2007, Section 6.5, Carloni, Sanfelice, Teel, & Melchiorri, 2007 and Tarn, Wu, Xi, & Isidori, 1996). The second application pertains to the bouncing ball system (Guckenheimer & Holmes, 1983) with a control input affecting the impacts (see also Brogliato & Rio, 2000, Galeani, Menini, Potini, & Tornambé, 2008, Menini & Tornambe, 2001, Sanfelice, Teel, & Sepulchre, 2007, Tornambé & Menini, 2003 and Zavala-Rio & Brogliato, 1999 where stabilization and, respectively, trajectory tracking for the so-called *juggling systems*, namely mechanical systems controlled at impacts, has been addressed, and Leine & Heimsch, 2012 where the stability of a controlled bouncing ball system is studied using Lyapunov-like techniques). Classical passivity-based control techniques such as *passivation* by feedback, *energy shaping* and *damping injection* are also applied to the two applications to illustrate their effectiveness in the hybrid systems setting.

1.3. Organization

The remainder of the paper is organized as follows. In Section 2, the two driving applications are presented. Section 3 introduces definitions of passivity and conditions to link these properties to asymptotic stability. In Section 4 a passivity-based control result is given and then applied to the special passivity cases of the two applications. The obtained passivity-based controllers are validated via simulations in Section 5.

2. Motivational applications

In this paper, the two applications shown in Fig. 1 drive the study of passivity and passivity-based control for hybrid systems.

2.1. Application 1: a point mass interacting with the environment

We consider the mechanical system depicted in Fig. 1(a), which consists of a point mass driven by a controlled force. The mass is constrained to move horizontally and, during its motion, it may come into contact with a surface located at the origin of the line of motion. The position and the velocity of the mass have been denoted with x_1 and x_2, respectively.

When the impact velocity is lower than a certain threshold, denoted as $\bar{x}_2 > 0$, a compliant impact model is adopted (Stronge, 2000). Assuming unitary mass for the sake of simplicity, the system is described by the following equations:

$$\dot{x}_1 = x_2, \qquad \dot{x}_2 = v_c - f_c(x), \tag{1}$$

where $v_c \in \mathbb{R}$ denotes the steering input, f_c the contact force given by

$$f_c(x) = \begin{cases} k_c x_1 + b_c x_2 & \text{if } x_1 > 0 \\ 0 & \text{if } x_1 \le 0 \end{cases}$$

in which $k_c > 0$ and $b_c > 0$ are, respectively, the elastic and damping coefficients of the compliant contact model.

When a collision with the surface occurs with a velocity of the mass greater or equal than $\bar{x}_2$, possible changes in the contact