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Phase synchronization control of complex networks of Lagrangian systems on adaptive digraphs[☆]

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ABSTRACT

This paper presents a formation control and synchronization method that utilizes adaptive network topologies for a class of complex dynamical networks comprised of a large number of highly-nonlinear Euler–Lagrange (EL) systems. A time-varying and switching network topology, constructed by the adaptive graph Laplacian matrix, relaxes the standard requirement of consensus stability, even permitting exponential synchronization on an unbalanced digraph or a weakly connected digraph that can sporadically lose connectivity. The time-varying graph Laplacian matrix is adapted by an adaptive control scheme based on relative positions and errors of synchronization and tracking. The adaptive graph Laplacian is integrated with a phase synchronization controller that synchronizes the relative motions of EL systems moving in elliptical orbits, thereby yielding a smaller synchronization error than an uncoupled tracking control law in the presence of bounded disturbances and modeling errors. An example of reconfiguring hundreds of spacecraft in Low Earth Orbit shows the effectiveness of the proposed phase synchronization controller for a large number of complex EL systems moving in periodic elliptical orbits.

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1. Introduction

This paper is concerned with complex dynamical networks comprised of nonlinear Euler–Lagrangian (EL) systems, which often represent the dynamics of robots and spacecraft more accurately. Because of the nonlinear terms of an EL system, such as nonlinearly-coupled inertia matrices, sophisticated nonlinear control methods along with rigorous stability proofs should be used. Another aspect of complexity considered in this paper is the dimensionality or structural complexity of networks. The present paper is motivated by such a real-world problem of controlling and reconfiguring a large number of spacecraft (100 s–1000 s) in the presence of external disturbances (Chung & Hadaegh, 2011; Morgan et al., 2012). The objective of this paper is to study the problem of generating stable adaptive network topologies that can reduce the complexity of controlling a large network comprised of coupled EL systems.

1.1. Problem statement and contribution

Let us visualize multiple rigid bodies following some periodic orbit as shown in Fig. 1. Oftentimes, it is more useful to maintain a formation shape by enforcing some position (phase) differences than exactly following a desired trajectory (e.g., synthetic apertures in space (Chung & Hadaegh, 2011)). Hence, this paper solves the problem of combining phase synchronization with tracking control for a large number of coupled EL systems. Tracking control with exponential stability ensures that each EL system ($\mathbf{q}_j$) yields bounded errors with respect to its desired trajectory or (virtual or real) leader $\mathbf{q}_{d,j}(t)$ in the presence of bounded disturbances and modeling errors. Phase synchronization in this paper means maintaining a desired phase difference between neighboring EL systems. Then, the objective is to ensure that the phase synchronization errors are smaller than tracking errors, resulting in

$$\lim_{t\to\infty} \|\mathbf{q}_j - \mathbf{f}(\mathbf{q}_k, \phi)\| < \lim_{t\to\infty} \|\mathbf{q}_j - \mathbf{q}_{d,j}\| \le \Delta, \quad \exists \Delta > 0 \tag{1}$$

where $\mathbf{f}(\mathbf{q}_k, \phi)$ rotates the position of a neighbor ($\mathbf{q}_k$) by the angle ϕ. This paper presents a strategy of making phase synchronization ($\mathbf{q}_j \to \mathbf{f}(\mathbf{q}_k, \phi)$) occur faster than tracking ($\mathbf{q}_j \to \mathbf{q}_k$) on an adaptive graph.

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The standard synchronization problem ($\mathbf{q}_j \rightarrow \mathbf{q}_k$) of EL systems has been studied without tracking control (Chopra & Spong, 2006; Ren, 2009). However, such a controller without a leader or desired trajectory might result in undesirable drifting of the synchronized states. The exact synchronization ($\mathbf{q}_j = \mathbf{q}_k$ and $\phi = 0$) of EL systems, combined with trajectory tracking, was studied first by Rodriguez-Angeles and Nijmeijer (2004). However, their work did not consider network graphs that permit other communication topologies than all-to-all coupling, which is a prohibitive requirement for a large network. Moreover, the controller required acceleration terms. In contrast, our prior work (Chung & Slotine, 2009) presented a generalization of the combined synchronization and tracking control of EL systems on a network graph. Tracking control of a (virtual) leader adds another layer of robustness to multi-agent consensus in case the network graph loses connectivity sporadically. Recent papers followed the combined tracking and synchronization framework for networked EL systems (e.g., Dong (2011), Liu and Chopra (2010) and Nuno, Ortega, Basañez, and Hill (2011)). However, the aforementioned prior work neither considered the problem formulation (1) in which phase synchronization takes precedence over tracking nor studied an adaptive network topology.

Why directed graphs? In this paper, directed graphs (digraphs) are preferred over undirected graphs since each directed edge realistically represents a heterogeneous capability of communication or relative sensing (Smith & Hadaegh, 2007) of each member. Whereas much of prior work used undirected graphs for their network models, real-world complex networks (e.g., biological circuits) are mostly directed graphs (Liu, Slotine, & Barabási, 2011). However, all the results in this paper hold vacuously for undirected graphs since they are balanced digraphs. Next, we state the main contributions of this paper.

Contribution 1: *Adaptive graph Laplacian.* The EL systems are coupled through a diffusive term (e.g., $\mathbf{q}_j - \mathbf{q}_k$) in each controller, whose coupling gains are computed by an adaptive control law. Then, the information flow in the network is epitomized by the adaptive graph Laplacian matrix, whose element is either nonzero or zero, depending on whether there is a directed communication link between each pair of the agents. For a large network, the graph Laplacian matrix tends to be sparse. Most prior work used a constant or switching Laplacian matrix, comprised of fixed gains for each network topology. Consensus stability of linear systems on time-varying network topologies were investigated in Hong, Hu, and Gao (2006), Ji and Egerstedt (2007), Kim and Mesbahi (2006) and Ren and Beard (2005). Another prior work studied the pose synchronization of rigid bodies on switching networks (Hatanaka, Igarashi, Fujita, & Spong, 2012). Also, random graphs (Hatano & Mesbahi, 2005; Tahbaz-Salehi & Jadbabaie, 2010) could be employed to handle a large network. However, most prior work studied the effects of switching or time-varying topologies on the consensus stability without suggesting a systematic method of automatically determining the network topology of a large complex network.

In contrast with prior work, this paper uses an adaptive control scheme to automatically compute a time-varying network topology. In other words, the adaptive graph Laplacian method determines not only which neighbors each member should communicate with, but also the time-varying elements of the Laplacian matrix (Chang, Chung, & Blackmore, 2010; Chung, Chang, & Hadaegh, 2011). Hence, the proposed adaptive Laplacian differs from the method of adapting the scalar coupling strength of a fixed graph used in Yu, Chen, and Lu (2009). Further, the use of integration-based adaptation, based on relative positions and synchronization/tracking errors, makes the proposed adaptive Laplacian differ from other methods of time-varying Laplacians that depend only on relative distances (Cucker & Smale, 2007; Ji & Egerstedt, 2007; Kim & Mesbahi, 2006). An integral adaptation law also results in smooth switching of network topologies.

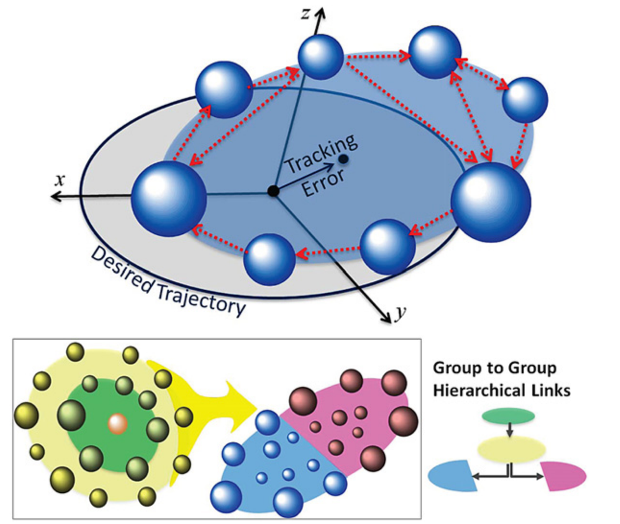

Fig. 1. (Top) Multiple EL systems shifted from the desired orbit show smaller synchronization errors than tracking errors. The dotted lines indicate directed communication links. (Bottom) Concurrent synchronization of multiple hierarchically-combined groups (see Section 4.3). The synchronization of the green group sends the synchronous desired input to the yellow group, whose synchronous trajectory in turn enters the red and blue groups. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

An adaptive network topology is useful if the network goes through numerous reconfigurations of the network topology. A large number of agents in a network can be effectively synchronized and stabilized without constraining the network to predefined stabilizing topologies like balanced digraphs (Olfati-Saber & Murray, 2004) or strongly connected graphs (Liu & Chopra, 2010). Another benefit of the proposed adaptive scheme is that inverse-optimality (Krstic & Li, 1998; Luo, Chu, & Ling, 2005) can be used to design an adaptive control law, without solving a computationally-expensive online optimization scheme. Also, we will show that the required gain for stabilization is smaller by employing a projection-based adaptive scheme. Furthermore, the technique of hierarchically combining two types of inputs (i.e., tracking and synchronization) can be used in tandem with the adaptive Laplacian method to achieve concurrent synchronization among numerous, hierarchically-divided subgroups (see Fig. 1).

Contribution 2: *Nonlinear stability analysis of a hierarchically combined, adaptive networks.* The proposed adaptive Laplacian method is presented with nonlinear stability proofs for achieving the control objectives in (1). In fact, the stability results in this paper are derived to allow many different choices of adaptation laws. For stability analysis of networked nonlinear systems, the passivity of the input–output dynamics can be exploited (Arcak, 2007; Chopra & Spong, 2006; Hatanaka et al., 2012; Ihle, Arcak, & Fossen, 2007). Input-to-state stability (ISS) is also useful to study stability of networked systems with bounded uncertainties (e.g., Nešić and Teel (2004) and Rüffer, Kellett, and Weller (2010)). In this paper, contraction analysis (Lohmiller & Slotine, 1998), which has been successfully applied to networked dynamics (Chung, Ahsun, & Slotine, 2009; Chung & Slotine, 2009, 2010; Pham & Slotine, 2007; Wang & Slotine, 2006a), is used to analyze the synchronization stability and robustness of the virtual observer-like systems (Wang & Slotine, 2005) that are constructed by the proposed controller. We will show that if an unperturbed virtual system is contracting (i.e., globally exponentially stable), then the virtual system with a bounded disturbance is ISS and finite-gain $\mathcal{L}_p$ stable with $p \in [1, \infty]$. Many types of model uncertainty can be cast into a

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