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Nonparametric identification of a Wiener system using a stochastic excitation of arbitrarily unknown spectrum ☆

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ABSTRACT

A Wiener system consists of two sequential sub-systems: (i) a linear, dynamic, time-invariant, asymptotically stable sub-system, followed by (ii) a nonlinear, static (i.e. memoryless), invertible sub-system. Both sub-systems will be identified non-parametrically in this paper, based on observations at only the overall Wiener system's input and output, with*out* any observation of any *internal* signal inter-connecting the two sub-systems, and with*out* any prior parametric assumption on either sub-system. This proposed estimation allows the input to be temporally *correlated*, with a mean/variance/spectrum that are a priori *un*known (instead of being white and zero-mean, as in much of the relevant literature). Moreover, the nonlinear sub-system's input and output may be corrupted additively by Gaussian noises of non-zero means and *un*known variances. For the above-described set-up, this paper is first in the open literature (to the best of the present authors' knowledge) to estimate the linear dynamic sub-system non-parametrically. This presently proposed *linear* system estimator is analytically proved as asymptotically unbiased and consistent. Moreover, the proposed *non*linear sub-system's estimate is assured of invertibility (unlike earlier methods), asymptotic unbiasedness, and pointwise consistence. Furthermore, both sub-systems' estimates' finite-sample convergence is also derived analytically. Monte Carlo simulations verify the efficacy of the proposed estimators and the correctness of the derived convergence rates.

1. Introduction

1.1. Why Wiener system?

A Wiener system consists of (i) a linear, dynamic, causal, time-invariant, asymptotically stable sub-system, followed by (ii) a nonlinear, memoryless (static), invertible sub-system. A block diagram of the Wiener system is shown in Fig. 1, showing the two sub-systems in cascade, and defining various symbols' inter-relationship.

The Wiener system model has been used in diverse applications: to model the inter-relationship between muscle tension and muscle length [6], the human visual system [9], the propagation channel in satellite microwave communications [14], or power amplifiers [29].

1.2. Why nonparametric modeling of the nonlinearity?

Wiener system identification may be traced to the 1970s, but this decades-long literature mostly assumes the to-be-estimated nonlinearity to follow some prior known functional form, typically a polynomial of a prior known

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0165-1684/

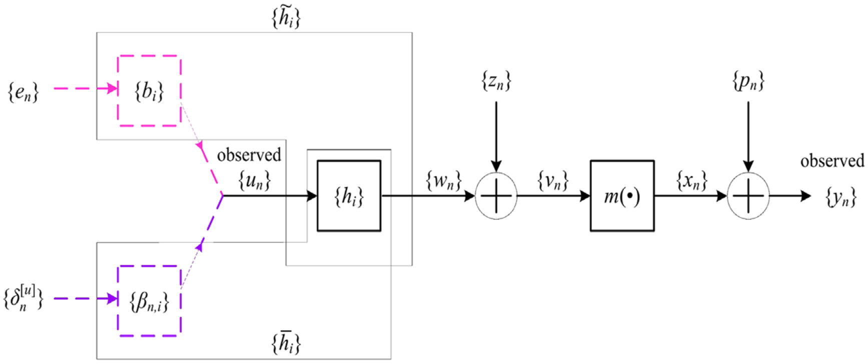

Fig. 1. The schematic of a Wiener system, with disturbance $\{p_n\}$ corrupting the Wiener system's output.

order or a piecewise-linear function. The system-identification problem then degenerates to a parameter-estimation problem to estimate the parameters of the presumed model.

Such parametric presumption is rigid and could inherently lead to modeling biases. What if the presumed model is invalid? What if no prior information is available to make any presumption?

*Non*parametric modeling, in contrast, relaxes the restrictions implicit in any parametric model. *Non*parametric system identification thus requires little prior knowledge on the unknown nonlinearity, reflecting the reality that such prior knowledge is often unavailable in practice. *Non*parametric estimation algorithms are advanced in [13,15,17,27,19,20,22,24,31] for Wiener models. This work will allow the static nonlinearity to *non* parametrically be any Borel-measurable function that is invertible, piecewise twice-differentiable, Lipschitz continuous, and bounded.

1.3. Why stochastic input signal?

A physical system is often subject to stochastic influences, besides deterministic influences. Or, the system inputs' mathematical models may be unknown, and thus to be regarded conveniently as a stochastic time series. This results in the scenario of a deterministic system excited by purely stochastic inputs, as in [13,17,19,20,22,24,25,31].

This work models the Wiener system's input signal as random, Gaussian-distributed, and possibly colored with an unknown frequency-spectrum. Moreover, this work allows the possibility of the Wiener system being randomly perturbed between the two sub-systems, and/or at the nonlinear static sub-system's output before the observations are collected. These random perturbations are each modeled as white Gaussian noise, at a (possibly) non-zero mean (that is constant and prior known) and at a possibly unknown (but constant) variance.

1.4. Why spectrally colored input signal?

The practicing engineer often has only limited physical leeway with the instrumentation to physically excite a specific time–frequency signal as an input to the Wiener system – recalling that the excited "signal" is often not simply an electric voltage or current, but a chemical, mechanical or thermal process. A spectrally colored times series could be physically easier (under some environments to generate) than a perfectly white times series.

Moreover, if the system is to operate only within a certain frequency-subband (e.g. a power amplifier transmitting a band-limited signal over a particular frequency-band), then the nonlinear subsystem's behavior at that relevant frequency-band can be more accurately estimated by using a spectrally colored input whose energy is concentrated in only that frequency-band, than by using a white input whose energy must necessarily be dispersed over all frequencies.

In the open literature of Wiener system identification via nonparametric means using a stochastic input, the *only* reference appears to be Greblicki's important contribution in [22]. This present work (i) will improve on that seminal work's nonlinear subsystem estimation accuracy and convergence, (ii) will guarantee the nonlinearity estimate of invertibility, (iii) will estimate also the *linear* dynamic subsystem (which is not estimated in [22]) in parallel to the estimation of the *non*linear static subsystem, (iv) will allow the linear dynamic subsystem of non-causality, (v) will tolerate the input of any non-zero mean (that may remain *un*known to the estimators), and (vi) will permit the unobserved perturbations of any zero/nonzero mean.

1.5. Contribution of this work

Referring to the notation in Fig. 1, this paper will non-parametrically estimate the linear dynamic sub-system's impulse response $\{h_n, n=-Q, 1-Q, \ldots, -1, 0, 1, 2, \ldots\}$ and the nonlinear static subsystem's input–output nonlinearity $m(\cdot)$. Observable is only the input and the output of the overall Wiener system (i.e., $\{(u_n, y_n), \forall n\}$), but not any signal nor any noise (e.g. $\{p_n, v_n, w_n, x_n, z_n, \forall n\}$) internally interconnecting the two sub-systems. These two sub-systems' proposed estimators are algorithmically independent from each other, in the sense that each could be computed without computing the other, or that the two subsystems may be estimated simultaneously in parallel.

More specifically, this work will accommodate the following models of the nonparametric Wiener system, its colored input, and its unobservable perturbation: (i) the