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Delay-error-constrained minimax design of all-pass variable-fractional-delay digital filters

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ABSTRACT

This paper first derives a simplified variable-fractional-delay (VFD) expression for the all-pass (AP) VFD digital filter, and then uses the simplified VFD expression to formulate the minimax AP-VFD filter design as a two-step linear-programming (LP) problem. To suppress the maximum error of the VFD response (VFD-peak-error), this minimax design minimizes the maximum error of the variable-frequency-response (VFR) subject to the VFD-peak-error constraint. Thus, this two-step design can minimize the VFR-peak-error with the VFD-peak-error suppressed below a prescribed upper bound. With the aid of the simplified VFD expression, the VFD-peak-error constraint can be approximately linearized as a linear one, and thus the minimax design can be solved by using the LP method. We will use an illustrative example to verify that the simplified VFD expression is almost the same as the true one, and that the proposed LP-based two-step minimax design can significantly suppress the VFD-peak-error.

1. Introduction

The digital filters having tunable fractional delay responses are called variable-fractional-delay (VFD) filters [1]. In the past decades, VFD digital filters have received tremendous attention because they are useful in various digital-signal-processing fields. A typical application is discrete-time signal interpolation [2]. VFD filters are also applicable to timing-offset adjustments in digital communications, sampling-rate conversion, and time-delay estimation [1–3]. Two types of VFD digital filters can be designed: one is the non-recursive type [4–11], and the other is the recursive type. Among the non-recursive type of VFD filters, the so-called Lagrange-type VFD filters have closed-form impulse responses [2], while the general form of non-recursive VFD filters has their coefficients expressed as different polynomials in the VFD parameter. The optimal polynomial coefficients can be found in some error sense through utilizing various optimization approaches [4–11].

The main objective of this paper is to deal with a special type of recursive VFD filters that have unity-gain feature. Such recursive VFD filters are called all-pass (AP) VFD filters. For simplicity, we denote all-pass VFD filters as AP-VFD filters. Since AP-VFD filters have a unique feature, i.e., they have *unity-gain* response in the whole frequency domain, and the designs of AP-VFD filters have received tremendous attention in the past decades [12–22]. It should be noted that such a unity-gain feature cannot be exactly realized by using non-recursive VFD filters, although non-recursive VFD filters can only approximate the unity-gain response (just approximation). Here, we should emphasize that the unity-gain feature is mandatory in many practical situations where the gain must not exceed unity [3].

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So far, a wide variety of techniques have been developed for designing AP-VFD filters through exploiting the unity-gain feature. Most of the existing design methods minimize either the variable-frequency-response (VFR) error or the variable-phase-response (VPR) error in the weighted-least-squares (WLS) sense or in the minimax sense [14–22]. We have also pointed out that minimizing the VFR error is almost equivalent to minimizing the VPR error in the AP-VFD filter case. This has been proved and identified in [21]. In the minimax design [19], the peak error of the VFR (VFR-peak-error) is minimized. As a result, the VFR-errors are shaped to be considerably flat. This is the final goal of the minimax design [19]. On the other hand, the maximum group-delay error is not controllable because the minimax-design formulation does not impose any VFD-error constraints on the design. Consequently, the maximum VFD-error (VFD-peak-error) may become too large to be acceptable in practical applications. Therefore, it is desirable to constrain the VFD-peak-error such that the VFD-peak-error is kept below a prescribed value (threshold). This is the main objective of this paper.

This paper aims to minimize the VFR-peak-error of the AP-VFD digital filter while simultaneously suppressing the VFD-peak-error. Originally, the VFD response is non-linear with respect to the filter coefficients to be found. This implies that the design constraints on the delay error are inherently non-linear. Hence, the minimax design cannot be achieved by using the linear-programming (LP) method. In this paper, we first derive a simplified VFD expression (formula) for the AP-VFD filter. This simplified VFD expression is an approximated one whose numerator becomes completely linear with respect to the filter coefficients. By using this simplified VFD expression, we can obtain a simplified delay-error function and then formulate the minimax design that minimizes the VFR-peak-error subject to the VFD-peak-error constraint. This minimax design can be easily solved by utilizing the LP method.

The LP-based minimax design includes a two-step procedure. The first step (Step-1) is to achieve a rough minimax design without VFD-peak-error constraint, where the denominator of the VFR-error function is ignored. Therefore, this step leads to an initial design (rough design). Once the initial design is completed, the second step (Step-2) uses the denominator resulting from Step-1 to solve the LP problem with appended VFD-peak-error constraints. This Step-2 can significantly reduce the VFR-peak-error and further suppress the VFD-peak-error, which leads to an improved design (fine design). We will use illustrative examples to show the following two points:

(1) The simplified VFD expression is considerably consistent with the true one. This simplified formula enables the minimax design with VFD-peak-error constraint to be formulated as an LP problem.
(2) The LP-based two-step minimax design can significantly suppress both the VFR-peak-error and the VFD-peak-error as compared with other AP-VFD filter design techniques.

2. AP-VFD digital filter

In this section, we first briefly review the variable-frequency-response (VFR) of the AP-VFD filter [19,21], and then derive a simplified group-delay expression as well as its error function. The next section will utilize the VFR-error expression and the VFD-error expression to formulate the LP-based minimax design.

2.1. VFR and VFR-error function

The Nth-order AP-VFD digital filter has the transfer function with coefficients $a_n(p)$ as

$$H(z,p)=\frac{a_N(p)+a_{N-1}(p)z^{-1}+\cdots+a_1(p)z^{-(N-1)}+z^{-N}}{1+a_1(p)z^{-1}+\cdots+a_{N-1}(p)z^{-(N-1)}+a_N(p)z^{-N}}=z^{-N}\cdot\frac{D(z^{-1},p)}{D(z,p)} \tag{1}$$

where the denominator is

$$D(z,p)=\sum_{n=0}^{N}a_n(p)z^{-n} \tag{2}$$

with $a_0(p)=1$. The coefficients $a_n(p)$ are variable, and each of them is expressed as an Mth-degree polynomial in the VFD parameter p.

To reduce the filter complexity, we have proved in [15] that $a_n(p)$ should take the form

$$a_n(p)=\sum_{m=1}^{M}b_{nm}p^m \tag{3}$$

but not the form

$$a_n(p)=\sum_{m=0}^{M}b_{nm}p^m.$$

This form has been adopted in [15,19–22] for designing AP-VFD filters. Likewise, we also adopt it in the design formulation.

The objective of the AP-VFD filter design is to approximate the desired group-delay:

$$\tau_d(\omega,p)=N+p \tag{4}$$

by using the transfer function in (1), where N is the order of the AP-VFD filter, p is a fractional number in the range

$$p\in[p_{\min},p_{\max}]$$

with

$$p_{\max}=p_{\min}+1$$
$$p_{\min}=-0.5$$

and $\omega\in[0,\alpha\pi]$, $\alpha\in(0,1)$, is the interested frequency band.

It is clear that the desired VFD in (4) corresponds to the desired phase

$$\Theta_d(\omega,p)=-(N+p)\omega. \tag{5}$$

Furthermore, the desired VFD in (4) corresponds to the desired VFR

$$H_d(\omega,p)=e^{j\Theta_d(\omega,p)}=e^{-j(N+p)\omega}. \tag{6}$$

Here, we re-derive the VFR expression as well as its error function stated in [19,21] in a slightly different way.