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Adaptive signal decomposition based on wavelet ridge and its application

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A B S T R A C T

Signal decomposition is a widely-used approach for multicomponent signal processing. To improve the accuracy and anti-noise performance of multicomponent decomposition, this paper proposes a novel multicomponent signal decomposition method based on wavelet ridge extraction, called the Wavelet ridge signal decomposition (WRSD). A wavelet ridge extraction algorithm is introduced. We find that this algorithm can obtain the wavelet ridge of one component in a multicomponent signal and the initial scale will determine the wavelet ridge of which component is extracted. Since the instantaneous frequency obtained by wavelet ridge has small frequency fluctuation, low-pass filtering is used to increase the accuracy of instantaneous frequency estimation. With the improved instantaneous frequency, the synchronous demodulation method is used to separate the corresponding component from the signal composition. By repeating this process, all components can be adaptively and automatically obtained. This method is employed to analyze three typical simulated vibration signals and compared with Hilbet vibration decomposition and empirical mode decomposition. The comparison results demonstrate its superiority in decomposition accuracy and noise insensitivity. Finally, the proposed WRSD method is successfully applied to the diagnosis of a shaft misalignment fault and a gearbox wear fault.

1. Introduction

In nature and engineering, there are many multicomponent signals, and most of these signals are nonstationary. Various signal processing methods have been proposed to analyze the multicomponent signals, such as short time Fourier transform (STFT) [1], Wigner–Ville distribution (WVD) [2], wavelet scalogram [3], iterative Hilbert transform [4], etc. A better way to reveal the time-varying amplitude and frequency characteristic is to decompose the multicomponent signal into monocomponent signals firstly, and then do further analysis with these decomposed components. Signal decomposition technology has been successfully applied to various fields, such as mechanical fault diagnosis [5,6], system identification [7], image texturing [8], biological data processing [9], etc.

In the past three decades, signal decomposition has been a hot point for signal processing research, and many effective methods have been proposed. The discrete wavelet transform was early used to decompose the signal [10]. But the frequency resolution at each scale is coarse due to the dyadic time–frequency grid. To improve the time–frequency resolution, overcomplete wavelet transform has been widely researched. For example, higher density wavelet transform [11] and dense framelet transform [12] have been successfully

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applied to the analysis of mechanical faulty vibration signal. A powerful approach, named as empirical mode decomposition (EMD), was proposed by Huang et al. [13]. This method has been widely applied to various fields including mechanical engineering, structural engineering, biomedicine, etc. However, EMD has the problem of mode mixing [14], which limits its frequency resolution. Recently, another multi-component decomposition method, called Hilbert vibration decomposition (HVD), was proposed by Feldman [15]. HVD has good frequency resolution and can distinguish various narrowband components while EMD cannot. However, both EMD and HVD are sensitive to additive random noise. They cannot extract low level signals from the large amplitude additive noise [16]. Like HVD, analytical mode decomposition (AMD) based on Hilbert transform was developed by Chen and Wang [17], which can separate closely spaced frequency components. Generalized demodulation time–frequency method and its improved methods are also proposed to process nonlinear multicomponent signals [18–20]. These methods have fine resolution, but it is very important to select the appropriate phase functions. More recently, sparse decomposition based on different transform basis was proposed by Qin [21], which has high decomposition accuracy, but its computation speed is relatively slow. We usually expect that the signal decomposition method is automatic, adaptive, accurate and of low noise sensitivity. With this motivation, a new multicomponent signal decomposition approach based on wavelet ridge is explored in this paper.

It is well-known that wavelet transform has good time–frequency localization property. Via wavelet ridge, the signal frequency can be well obtained. An adaptive wavelet ridge computation algorithm was proposed by Delprat et al. [22]. This algorithm is mainly used to analyze monocomponent signals, whereas it can also extract the wavelet ridge of one component of a multicomponent signal, and we find that the initial scale will determine the wavelet ridge of which component is extracted and its theoretical explanation is given. After obtaining one component's instantaneous frequency, the classic synchronous demodulation technique is used to reconstruct this component. Then this component is subtracted from the signal composition. By repeating this process, all components can be extracted. The decomposition process is adaptive, automatic and can obtain the high frequency component successively. In this study, the proposed method is named as Wavelet ridge signal decomposition (WRSD). Compared with commonly-used multicomponent decomposition methods, the proposed WRSD method has higher decomposition accuracy and is less sensitive to random noise with low intensity. The application results also show that the proposed method can better extract the weak fault feature from the mechanical vibration signals.

2. Wavelet ridge theory

2.1. Asymptotic signal and its exponential model

Let us first briefly revisit the basic definitions and properties of asymptotic signals. An arbitrary real energy signal $s(t)$ can be represented in terms of instantaneous amplitude $A(t)$ and phase $\phi(t)$, i.e. in the following form:

$$s(t) = A(t)\cos(\phi(t)) \tag{1}$$

where $A(t) \geq 0$ and $\phi(t) \in [0,\ 2\pi]$. Obviously, such a representation is not unique. Via Hilbert transform, it is convenient to specify a particular one. The analytic signal $Z_s(t)$ associated with $s(t)$ is obtained by Hilbert transform, which can be expressed as

$$Z_s(t) = (1 + iH)s(t) = A_s(t)\exp\left(i\phi_s(t)\right) \tag{2}$$

where H denotes the Hilbert transform.

Particularly assume that $s(t)$ is asymptotic, which essentially means that

$$\left|\frac{d\phi}{dt}\right| \gg \left|\frac{1}{A}\frac{dA}{dt}\right| \tag{3}$$

Then the instantaneous frequency of $s(t)$ can be calculated as

$$f_s(t) = \frac{1}{2\pi}\frac{d\phi_s}{dt} \tag{4}$$

2.2. Ridge and wavelet curves

By wavelet transform, we can also describe the time-varying frequency characteristic of the signal. Assuming that the signal and the wavelets are all asymptotic, with the stationary phase method, we can get some particular sets of curves in the time-scale half-plane, namely the ridge and the wavelet curve. The ridge describes instantaneous frequency characteristic of the signal.

Let $\psi(t)$ be an asymptotic analytic wavelet [22], which is expressed as

$$\psi(t) = A_\psi(t)\exp[i\phi_\psi(t)] \tag{5}$$

With this mother wavelet, the wavelet transform of $Z_s(t)$ is given by

$$\begin{aligned} W_Z(a,b) &= \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} Z_s(t)\psi^*\left(\frac{t-b}{a}\right)dt \\ &= \frac{1}{\sqrt{a}}\int_{-\infty}^{\infty} A_{a,b}(t)\exp\left[i\phi_{a,b}(t)\right]dt \end{aligned} \tag{6}$$

where a and b are respectively scale parameter and shift parameter, $\phi_{a,b}(t)$ and $A_{a,b}(t)$ are respectively given by

$$\phi_{a,b}(t) = \phi_s(t) - \phi_\psi\left(\frac{t-b}{a}\right) \tag{7}$$

$$A_{a,b}(t) = A_s(t)A_\psi\left(\frac{t-b}{a}\right) \tag{8}$$

The stationary point t_s is defined as $\phi'_{a,b}(t_s) = 0$, i.e.

$$\phi'_s(t_s) = \frac{1}{a}\phi'_\psi\left(\frac{t_s - b}{a}\right) \tag{9}$$

We can see from Eq. (9) that the stationary point t_s is the function of (a, b).

Then the ridge is defined to be the set of points (a, b) such that $t_s(a, b) = b$. It immediately follows from Eq. (9) that on the ridge

$$a = a_r(b) = \frac{\phi'_\psi(0)}{\phi'_s(b)} \tag{10}$$